Disclosure, Learning, and Coordination

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Abstract

We analyze how informed investors can learn from each other through disclosed trades. We show that disclosure always increases market efficiency but its effect on informed investors' profits is ambiguous. When informed investors have highly complementary signals, disclosure makes them coordinate their trades, so their expected profits are higher. Moreover, an informed investor with very imprecise information would prefer competition in the presence of disclosure as he learns more from the other informed investor than the market maker and makes more profits than he would obtain if he is the only informed investor in the market. As a result, when information acquisition is costly and endogenous, there could exist herding in information acquisition.

JEL classification: G12, G14, G18, G19.

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Disclosure of trades play an important role in financial markets. Earlier studies have found that disclosure can enhance market efficiency, increase market liquidity, and reduce profits of informed investors.[1] These studies typically assume that there is only one informed investor and thus ignore the effect of disclosure on mutual learning and coordination among informed investors. When there are multiple informed investors, they not only compete but also learn from each other. For investors with complementary signals, trade disclosure can act as a communication device that affects learning among informed investors differently from that of the public. The effects of disclosure on informed investors' strategy are complicated. In the presence of more multiple informed investors, how would disclosure affect market efficiency and liquidity? How would it affect learning and competition among informed investors? Will disclosure always decrease informed investors' profits? While it is sensible that disclosure will reduce informed investors' profits when they have highly correlated private signals, it is less clear otherwise.

In particular, we consider a model in which there is a risky asset and a risk free asset. At time zero, a public signal is announced. The public signal is the sum of the value of the risky asset and two noises. There are two informed investors, each knows one of the noises in the public information and thus is able to obtain an informational advantage over the public. The informed investor can be viewed as stylized fund managers with special insights about the public signal. If the two informed investors can combine their information, then they will have perfect information about the value of the risky asset. Informed investors trade with the market maker and liquidity traders. In the case of disclosure, we assume that informed investors have to disclose their trades immediately afterwards. In discrete time, we derive a recursive formula for the equilibrium, which can be solved by numerical methods. When the trading interval goes to zero, we derive a closed-form formula for the equilibrium. To determine the impact of disclosure, we compare our closed-form equilibrium formula with that obtained in Back, Cao, and Willard (2000, BCW), whose model is the same except without disclosure.

Disclosure of informed investors' trades creates incentives for informed investors to manipulate and they sometimes trade against their own valuation to mislead the market, so that the market maker cannot perfectly infer information from their trades. As a result, the informed investors randomize to manipulate the market maker's belief until the last moment of trading. The mixed strategy allows informed investors to maintain an informational ad-

[1]See Huddart, Hughes and Leving (2001), for example.

vantage over the market for a longer period of time. We show that the combined random components in informed investors' trade equals in distribution to that of liquidity traders. This is intuitively appealing as informed investors and liquidity traders will each contribute to half of the trading volume. Too much randomization will cause informed investors to lose a lot from randomized trade and too little randomization will cause informed investors to lose their informational advantage too early. To camouflage themselves, informed investors randomize such that their combined trading volume equals that of liquidity traders'.

The effects of trade disclosure on market efficiency is unambiguous. Market is more efficient at all times after disclosure. As informed investors know more about each other's signals, their valuations converge more quickly and they trade more aggressively on their information, which in turn makes the market more efficient.

On the contrary, the effect on market liquidity is ambiguous. Randomization will reduce the informational content in the aggregate order flow and thus increase market liquidity. However, learning among investors could reduce market liquidity. The reduction of asymmetric information would increase market liquidity. As a result, market liquidity can either increase or decrease depending on the parameters and the timing of the trades.

Similarly, the effects on expected profits of informed investors and market liquidity are more complicated. Public disclosure has three effects on informed investors' expected profits. The first is the randomization effect. As informed investors manipulate and add noise to their own trades, they lose money, which reduces their expected profits. The second is the learning effect. With trade disclosure, the aggregate informed trades serves as a sufficient statistic for the public. As each informed investor knows the random component in their own trades, informed investors learn twice more from disclosure than the public, which in turn increases their expected profits. The third is the market efficiency effect. Disclosure increases market efficiency, which reduces expected profits of informed investors.

When informed investors have very precise signals, their signals are substitutes and they won't be able to learn from each other as much. In this case the learning effect will be less important and disclosure decreases expected profits of informed investors. Without disclosure, when investors have very noisy signals, they tend to wait until they know more from each other before they trade aggressively. Trade disclosure can reduce the incentive to wait and make investors trade more aggressively. Informed investors learn more from disclosure than the market maker as they know their own endogenous random trades. The coordination effect could dominate other effects and result in higher expected profits of informed investors.

Learning from each other implies the possibility of herding. To analyze herding formally, we expand the model to two risk asset with symmetric identical and independent distributions. In the expanded model, there are two risky assets and one risk free asset. Each risky asset will have a public signal announced at time zero, and the public signal is the sum of the risky asset value and two independent noise terms. An informed investor who decides to investigate one risky asset will have access to know about one of the noise term and we assume that if both informed investors decides to investigate the same risky asset, they will have access to different noise terms in the public signal. We show that when informed investors have very noisy signals, they will prefer to acquire information on the same risky asset, so as to learn from each other through disclosure. In the presence of disclosure, an informed investor could prefer to have competition and thus disclosure facilitates herding in information acquisition. Interestingly, without disclosure, herding will never happen and informed investors will prefer to investigate different risky assets.

Interestingly, disclosure also makes informed investors trade collusively when they have conditionally uncorrelated signals. In this case, investors coordinate implicitly as their combined trades are the same as a monopolistic informed investor who has all the signals in the market. With conditionally uncorrelated signals, investors learn to become collusive and their combined trading strategy converges to that of a monopolistic investor as they get closer to the end of trading.

We extend the model to more than two informed investors. In this case, the gains from learning by informed investors over that of the market maker is reduced as now each informed investor only knows $1/N$ of the randomized noise trades. As a result, disclosure always reduces informed investors' profits. Nevertheless, with disclosure, it is still possible for an informed investor to make more profits in an oligopolistic setting than what he would receive in a monopolistic setting when the number of informed investors is strictly less than five and they have very imprecise information. Moreover, removing one informed investor from trading in the market can make the rest of informed investors worse off. Informed investors still herd in their decision to acquire information.

The most related research is Huddart, Hughes, and Levine (2001, HHL), which studies disclosure effect in a discrete-time Kyle model with a monopolistic informed investor. They show that the informed investor uses a mixed strategy in which the informed investor attaches a random order flow, for hiding information, to the information-based flow that is exactly the same as that in Kyle's model. In addition, mandatory disclosure unambiguously reduces informed investor's profits, increases market liquidity, and improves market efficiency. Since there is only one informed investor, there is neither learning nor competition among informed investors.[2] The effect of disclosure rules on informed investors' trading has also been studied by a number of authors including Fishman and Hagerty (1995) and John and Narayanan (1997). Fishman and Hagerty (1995) study a two-period model when an informed investor only possesses inside information with a certain probability. While an *informed* informed investor will never manipulate the market in their model, an *uninformed* informed investor can manipulate the market since the market may mistakenly believe that the *uninformed* informed investor is *informed*.[3]

The rest of the paper is organized into sections as follows. The model is described in Section 1. Section 2 discusses the condition for equilibrium with public disclosure in a discrete-time framework and offers a closed-form formula for the equilibrium in the limit when trading periods goes to infinity. Section 3 gives comparative statistics such as the effects of the correlation of private signals and disclosure on the intensity of trading, the rate of information transmission, the depth of the market, and the expected profits of informed investors. Section 4 studies how the correlation of private signals affect the market depth, and the expected profits of informed investors. Section 5 extends the model from a duopolistic setting to a general multiple players setting. Section 6 concludes. All proofs are left to the appendices.

1 The Model

We consider an economy with two informed investors who are required to disclose their trades based on the classic model of Kyle (1985). In our model, there is one risk-free asset

[2] Gong and Liu (2012) extend their results to multiple informed investors. In their model, informed investors have homogeneous information and thus as trading frequency goes to infinity, information will be revealed in opening trades and the expected profits for informed investors go to zero. Zhang (2004) shows that when the informed investor is risk averse, trade disclosure can reduce market efficiency as the risk-averse investor will be facing less price risk in the future when he unloads his positions and thus will not trade in a hurry.

[3] In models with disclosure but with multiple trading periods, Chakraborty and Yilmaz (2004) show that when the market faces uncertainty about the existence of the insider in the market and when there is a large number of trading periods before all private information is revealed, long-lived informed investors will manipulate in every equilibrium. Brunnermeier (2005) shows how disclosure of intermediary public information can cause investors with short-term noisy information to manipulate the market.

and one risky asset. An announcement is made at time 1 that reveals the liquidation value of the risky asset. The risk-free rate is taken to be zero. There are 2 risk-neutral informed investors and many liquidity traders who trade for liquidity reasons. Trading takes place over time interval [0, 1). In the discrete-time version of the model, there are M periods over time [0,1), and the time between any two consecutive trading periods is $\Delta t = 1/M$.

Let v denote the liquidation value of the risky asset at time 1. At time zero, there is a public signal q about the stock value:

$$q = v - \sum_{1}^{2} s^i. \tag{1}$$

Prior to observing the signal q, the correlation of s^1, s^2 is θ. The variance of s^i is the same across i and is denoted σ_s^2, s^i is independent of v. Informed investor i observes the noise in the public signal, s^i. In such a setting, each informed investor has information advantage over the market as they know how to interpret the public signal better.

Given q, we can rewrite the asset value as

$$v = q + \sum_{1}^{2} s^i. \tag{2}$$

Therefore given the public signal q, the stock value is a sum of the signals of the informed investors. Moreover, it is straightforward to show that as σ_s^2 goes to infinity, the conditional correlation of s^i given public information q goes to -1, and as σ_s^2 goes to zero, the conditional correlation of s^i goes to θ.

We use ρ_0 to denote the conditional correlation coefficient of s^1 with s^2 given q. We have

$$\rho_0 \equiv \text{corr}[s^1, s^2|q] = \frac{\theta\sigma_v^2 - (1-\theta^2)\sigma_s^2}{\sigma_v^2 + (1-\theta^2)\sigma_s^2}$$

In addition, we have

$$\Sigma_0 \equiv \text{var}[v|q] = \frac{2(1+\theta)\sigma_s^2\sigma_v^2}{\sigma_v^2 + 2(1+\theta)\sigma_s^2} \tag{3}$$

Let

$$\epsilon \equiv s^1 - s^2; \quad \Sigma_\epsilon \equiv \text{var}[\epsilon] = 2(1-\theta)\sigma_s^2$$

We can rewrite the signals investors have as

$$s^1 = \frac{-q + v + \epsilon}{2}, s^2 = \frac{-q + v - \epsilon}{2}$$

Then we have

$$\rho_0 = \frac{\Sigma_0 - \Sigma_\epsilon}{\Sigma_0 + \Sigma_\epsilon}$$

In the special case of $\theta = 1$, then $\rho_0 = 1$, each informed investor has perfect information about v. For convenience, we introduce the following notation

$$\delta_0 \equiv \frac{\mathrm{var}[v|q] - \mathrm{var}[v|s^1, q]}{\mathrm{var}[v|q]} = \frac{\mathrm{var}^{-1}[v|s^1, q] - \mathrm{var}^{-1}[v|q]}{\mathrm{var}^{-1}[v|s^1, q]}. \tag{4}$$

This is a measure of the quality of private information of informed investor 1 and by the argument of symmetry, informed investor 2 as well. Specifically, δ_0 is the "R-squared" in the linear regression of v on s^i for an arbitrary i, i.e., it is the percentage of the variance in v that is explained by a single informed investor's information. Alternatively, it is also the percentage drop in precision of informed investors to that of the market maker. It is easy to check that δ_0 is related to ρ_0 by the following equation

$$\delta_0 = \frac{1}{2} + \frac{1}{2}\rho_0 = \frac{\Sigma_0}{\Sigma_0 + \Sigma_\epsilon}. \tag{5}$$

Thus, when δ_0 is larger than, equal to, or smaller than 1/2, informed investors' signals are conditionally positively correlated, uncorrelated or negatively correlated respectively. When σ_s is small (large), each informed investor has very precise (coarse) information of the liquidation value.

In each trading period m, a risk-neutral market maker receives the total order from all informed investors and liquidity traders. Based on such order flow information, the market maker adjusts the price P_{m-1} to a new price P_m at which he buys or sells the risky asset to clear the market in period m. Since the market maker is assumed to be risk neutral, price P_m must be the conditional expectation given all public information. We use x_m^i to denote informed investor i's order, and use z_m^0 to denote the total order by all liquidity traders. We assume that z_m^0 are serially uncorrelated and normally distributed with mean zero and variance $\sigma_u^2 \Delta t$

$$E\left[z_m^0\right] = 0, \quad \text{and} \quad \mathrm{var}\left[z_m^0\right] = \sigma_u^2 \Delta t, \quad \text{for all } m.$$

For simplicity, we assume $\sigma_u = 1$. In addition, z_m^0 is independent of all other random variables in the model. Moreover, we assume that informed investors are prevented from any market-making activities, and hence when they submit their orders in period m they have no information about the mth-period order flow from any other party.

The only difference between a model with disclosure and a model without disclosure is whether or not each informed investor is required to disclose his mth period trade immediately after all trades are completed in period m. Technically, this implies the following difference in how each of the involved parties behaves in the model. Without disclosure, (1) the market maker sets his price P_m by observing the history of the aggregate order flow $\{y_k\colon 1 \leq k \leq m\}$, where

$$y_k \equiv z_k^0 + \sum_{1 \leq i \leq 2} x_k^i$$

and (2) each informed investor i decides his trade by observing his own past order flow $\{x_k^i\colon 1 \leq k < m\}$, his own signal s^i, and the past price history $\{P_k\colon 1 \leq k < m\}$. With disclosure, (1) the market maker sets his price by observing the breakdown of all traders' past order flow $\{x_k\colon 1 \leq k < m\}$ and $\{z_k^0\colon 1 \leq k < m\}$ together with the current aggregate order flow $\{y_m\}$; and (2) each informed investor i decides his trade by observing all traders' past order flow $\{x_k\colon 1 \leq k < m\}$ and $\{z_k^0\colon 1 \leq k < m\}$, in addition to his signal s^i and the past price history $\{P_k\colon 1 \leq k < m\}$. Note that in a model with disclosure, the breakdown of all the past order flow $\{x_k\colon 1 \leq k < m\}$ and $\{z_k^0\colon 1 \leq k < m\}$ are made public through public disclosure and price history.

The description above has focused on the discrete-time version of the model. An intuitive way to think of the continuous-time model is simply to take the limit of the discrete-time model with $M \to +\infty$.

2 The Solution

Under the disclosure requirement, informed investors announce their trades, $\{x_m^i\}$, $i = 1$, 2, immediately after the trade is executed. The market maker then adjusts his belief of the asset value from P_m (the market price for the risky asset in period m) to V_m, which is defined to be the market maker's estimate of the fair value of the risky asset with all the information up to and including the disclosure made at the end of period m. We can think of V_m as the pseudo-price that market maker would have set for the mth period trading if he had observed informed investors' orders before the execution of trades in the mth period. Although V_m is only a pseudo-price at which no trade ever takes place, it is important since it will be the starting point for the market maker to set P_{m+1} for the $(m+1)$th period of trading. In particular, in a linear equilibrium model that we will focus on, it is $P_{m+1} - V_m$ (as opposed to $P_{m+1} - P_m$) that will be linear to the total order flow submitted in the $(m+1)$th trading period.

Let $\underline{x}^i_m$ denote the history of investor i's trade in each past period before and including period m (i.e., $\{x^i_k \colon k = 1, \ldots, m\}$), let $\underline{y}_m$ denote the history of the net trade before and including period m (i.e., $\{z^0_k + \sum_{1 \le i \le 2} x^i_k \colon k = 1, \ldots, m\}$), and let $\underline{P}_m$ denote the price history before and including period m (i.e., $\{P_k \colon k = 1, \ldots, m\}$). With disclosure, informed investor i's private information prior to trading in period m includes his own signal s^i and the history of all past trades and prices $\underline{x}^1_{m-1}, \underline{x}^2_{m-1}, \underline{P}_{m-1}$. Let

$$x^i_m = x^i_m \left(s^i, \underline{x}^1_{m-1}, \underline{x}^2_{m-1}, \underline{P}_{m-1}\right)$$

represent the optimal strategy of informed investor i. Let

$$P_m = P_m \left(q, \underline{x}^1_{m-1}, \underline{x}^2_{m-1}, \underline{y}_m\right)$$

represent the optimal strategy of the market maker given the history of all orders and the current aggregate order.

Let X^i and P denote the strategy functions for informed investor i and the market maker, respectively. Given the strategy functions for informed investors and the market maker, the profit of informed investor i from trading in period m and on can be written as:

$$\pi^i_m(X^1, X^2, P) = \sum_{k \ge m} (v - P_k) x^i_k.$$

An equilibrium of the trading game exists if there is a 3-dimension vector of strategies, (X^1, X^2, P) such that:

1. For any $i = 1, 2$ and for all $m = 1, ..., M$, if $\hat{X}^i \neq X^i$,

   $$E\left[\pi^i_m(X^i, X^j) | q, s^i, \underline{x}^1_{m-1}, \underline{x}^2_{m-1}, \underline{P}_{m-1}\right] \ge E\left[\pi^i_m(\hat{X}^i, X^j) | q, s^i, \underline{x}^1_{m-1}, \underline{x}^2_{m-1}, \underline{P}_{m-1}\right]$$

   i.e., the optimal strategy is the best no matter which past strategies informed investor

i may have played.

2. For all $m = 1, \ldots, M$, we have

$$P_m = E\left[v \mid q, \underline{x}^1_{m-1}, \underline{x}^2_{m-1}, \underline{y}_m\right],$$

i.e., the market maker sets prices equal to the conditional expectation of the asset value given the order-flow history.

In this model, since investor i's trade at period m will be disclosed afterwards, the pricing and trading strategies for the no-disclosure case cannot be an equilibrium in the new setting. To see this, suppose the informed investor follows a strategy of [4]

$$x^i_m = \beta_m \Delta t s^i + L_1(\underline{x}^i_{m-1}) + L_2(\underline{x}^1_{m-1}, \underline{x}^2_{m-1})$$

where L_i, $i = 1, 2$ is a linear function of all public information. Then the market maker would infer

$$v = q + \frac{\sum_{1 \le i \le 2} \left[x^i_m - L_1(\underline{x}^i_{m-1}) - L_2(\underline{x}^1_{m-1}, \underline{x}^2_{m-1})\right]}{\beta_m \Delta t}$$

and choose

$$P_{m+1} = q + \frac{\sum_{1 \le i \le 2} \left[x^i_m - L_1(\underline{x}^i_{m-1}) - L_2(\underline{x}^1_{m-1}, \underline{x}^2_{m-1})\right]}{\beta_m \Delta t}$$

in the next period. Hence, in the next period, the market depth would be infinity. Understanding this, informed investors would have incentive to choose $\hat{x}^i_m \neq x^i_m$, which is inconsistent with the proposed equilibrium strategy.

We analyze a symmetric linear equilibrium. In particular, informed investor's trade can be written as

$$x^i_m = \beta_m \Delta t s^i + L_1(\underline{x}^i_{m-1}) + L_2(\underline{x}^1_{m-1}, \underline{x}^2_{m-1}) + z^i_m, \qquad (\star)$$

where (1) $\beta_m \Delta t s^i$ represents a private-information-based linear component, (2) $L_1(\underline{x}^i_{m-1}) + L_2(\underline{x}^1_{m-1}, \underline{x}^2_{m-1})$ is a public-information-based linear component, and (3) z^i_m is a noise component with z^i_m being normally distributed with mean 0 and variance $\sigma^2_m \Delta t$. Since informed investors are prevented from market-making activities, we further assume that z^i_m are independently distributed across agents. The market maker also uses linear rules for setting

[4] We restrict our attention to symmetric linear equilibria.

prices before disclosure and for updating his value estimate after disclosure. In particular,

$$P_m = V_{m-1} + \lambda_m \left(z_m^0 + \sum\nolimits_{1 \le i \le 2} x_m^i \right), \quad \text{and}$$
$$V_m = V_{m-1} + \bar{\lambda}_m \left(\sum\nolimits_{1 \le i \le 2} x_m^i \right).$$

The preceding equations imply that the random order from liquidity traders only has a temporary effect on price formation. In particular, liquidity traders' order in period m (i.e., z_m^0) only affects P_m but not P_k for any $k \ge m+1$: Once the mth-period disclosure is made, the market maker immediately abandons z_m^0 and adjusts his belief of asset value to V_m, which is not affected by z_m^0 and will be the base for forming future prices P_k ($k \ge m+1$).

Before stating our result, we first introduce some notation. Let F_m and F_m^i denote the information set of the market maker and informed investor i respectively *after* disclosure has been made in period m. Define

$$V_m \equiv \mathrm{E}[v|F_m], \qquad V_m^i \equiv \mathrm{E}[v|F_m^i],$$
$$\Sigma_m \equiv \mathrm{var}[v|F_m], \quad \Omega_m \equiv \mathrm{var}[v|F_m^i], \quad \text{and} \quad \delta_m \equiv \frac{\Sigma_m - \Omega_m}{\Sigma_m}.$$

THEOREM 1 *The necessary and sufficient conditions for a recursive linear symmetric equilibrium to exist are described below. For all $m = 1, \cdots, M-1$ and for informed investors $i = 1,\ 2$,*

$$x_m^i = \frac{\beta_m \Delta t}{2\delta_{m-1}} (V_{m-1}^i - V_{m-1}) + z_m^i \tag{6}$$

$$P_m = V_{m-1} + \lambda_m \left(z_m^0 + \sum_{i=1}^{2} x_m^i \right) \tag{7}$$

$$V_m = V_{m-1} + \bar{\lambda}_m \sum_{i=1}^{2} x_m^i \tag{8}$$

$$\sigma_m^2 = \beta_m \Sigma_m / (2\bar{\lambda}_m) \tag{9}$$

$$\lambda_m = \beta_m \Sigma_{m-1} / (\beta_m^2 \Delta t \Sigma_{m-1} + 1 + 2\sigma_m^2) \tag{10}$$

$$V_m^i - V_{m-1}^i = \frac{\Omega_{m-1} - \Omega_m}{\Omega_{m-1}} \left(v - V_{m-1}^i + \frac{z_m^j}{\beta_m \Delta t} \right) \tag{11}$$

$$V_m - V_{m-1} = \frac{\Sigma_{m-1} - \Sigma_m}{\Sigma_{m-1}} \left(v - V_{m-1} + \sum_{1 \le i \le 2} \frac{z_m^i}{\beta_m \Delta t} \right) \tag{12}$$

$$\Omega_m^{-1} = \Omega_{m-1}^{-1} + \beta_m^2 \Delta t/(\sigma_m^2) \tag{13}$$

$$\Sigma_m^{-1} = \Sigma_{m-1}^{-1} + \beta_m^2 \Delta t/(2\sigma_m^2) \tag{14}$$

$$E[\pi_m^i | F_{m-1}^i] = \alpha_{m-1}(V_{m-1}^i - V_{m-1})^2 + \zeta_{m-1} \tag{15}$$

$$\lambda_m = \alpha_m \bar{\lambda}_m^2 \tag{16}$$

$$\bar{\lambda}_m = \frac{2\lambda_m}{1 + \lambda_m \beta_m \Delta t(1 - 1/(2\delta_{m-1}))} \tag{17}$$

$$\alpha_{m-1} = \alpha_m \left(1 - \frac{\beta_m^2 \Delta t \Sigma_m}{2\sigma_m^2}\left(1 - \frac{1}{2\delta_{m-1}}\right)\right)^2 \tag{18}$$

$$\zeta_{m-1} = \zeta_m + \alpha_m \beta_m^2 \Delta t \left(\frac{\Omega_m}{\sigma_m^2} - \frac{\Sigma_m}{2\sigma_m^2}\right)^2 \left(\Omega_{m-1}\beta_m^2 \Delta t + \sigma_m^2\right) \tag{19}$$

subjecting to the boundary conditions

$$\beta_M = \sqrt{\frac{2\delta_{M-1}}{\Sigma_{M-1}\Delta t}}, \tag{20}$$

$$\lambda_M = \frac{\sqrt{2\delta_{M-1}\Sigma_{M-1}/\Delta t}}{1 + 2\delta_{M-1}}, \tag{21}$$

$$\alpha_{M-1} = \frac{1}{\lambda_M (1 + 2\delta_{M-1})^2}, \tag{22}$$

$$\zeta_{M-1} = 0, \tag{23}$$

and the second order condition

$$\lambda_m > 0. \tag{24}$$

COROLLARY 1 *In the special case that $\rho_0 = 0$, the model can be solved in closed form:*

$$\begin{aligned}
\lambda_m &= \sqrt{\Sigma_0}/2, & \beta_m &= M/[2\lambda_m(M - m + 1)], \\
\bar{\lambda}_m &= 2\lambda_m, & \sigma_m^2 &= (M - m)/[2(M - m + 1)], \\
\alpha_m &= 1/(4\lambda_m), & \Omega_m &= (1 - m/M)\Omega_0, \\
\zeta_m &= 0, & \Sigma_m &= (1 - m/M)\Sigma_0.
\end{aligned}$$

The results in the special case that $\rho_0 = 0$ are the same as the monopolistic model derived by HHL (2001). This is in sharp contrast to results on imperfect competition of informed investors without disclosure. Cao (1995), Foster and Viswanathan (1996), and BCW (2000) have shown that competition causes the market to be very illiquid and inefficient near the end of trade when there is no disclosure. With disclosure, we find that informed investors act in the aggregate as a monopolist when their signals are uncorrelated. In the uncorrelated-signal case, with disclosure, each informed investor knows his own random noise in the past. Consequently, each informed investor's conditional precision will remain to be twice of that of the market maker as they learn twice as fast. If informed investors' signals are uncorrelated to begin with, they remain conditionally uncorrelated due to public disclosure of trades after the fact. Therefore, disclosure makes informed investors coordinate with each other to maximize their profits and they act like a monopolist in the aggregate. On the contrary, without disclosure, BCW (2000) show that the conditional correlation coefficient of informed investors' signals goes to -1 even when the initial correlation coefficient is zero.

When the number of trading periods goes to infinity, the model approaches to the continuous-time model. Ignoring higher-order terms of Δt, we have the following proposition:

COROLLARY 2

$$\bar{\lambda}_m = \beta_m \Sigma_m \tag{25}$$

$$\lambda_m = \beta_m \Sigma_m / 2 \tag{26}$$

$$\sigma_m^2 = 1/2 \tag{27}$$

$$\bar{\lambda}_m = \frac{1}{2\alpha_m} \tag{28}$$

$$\frac{\Delta \Omega_m^{-1}}{\Delta t} = 2\beta_m^2 \tag{29}$$

$$\frac{\Delta \Sigma_m^{-1}}{\Delta t} = \beta_m^2 \tag{30}$$

$$\frac{\Delta \alpha_m}{\Delta t} = 2\alpha_m \beta_m^2 \Sigma_m \left(1 - \frac{1}{2\delta_m}\right) \tag{31}$$

$$\frac{\Delta \zeta_m}{\Delta t} = -\alpha_m \beta_m^2 (2\Omega_m - \Sigma_m)^2 / 2 \tag{32}$$

In the continuous-time model, we denote by $x(t)$, $t \in [0, 1]$ a variable x_m in the discrete-time model. For example, Σ_m in the discrete-time model is now denoted by $\Sigma(t)$. Taking the limit $\Delta t \to 0$, the difference equations above converge to a set of differential equations which leads to closed-form solutions described in Theorem 2.

THEOREM 2 *If $\theta < 1$, i.e., informed investors' signals are not perfectly correlated, as the number of trading rounds goes to infinity, in the limit we obtain*

$$\beta(t) = \frac{\sqrt{-\Sigma(t)'}}{\Sigma(t)} = \frac{1}{\sqrt{\Sigma_\epsilon}(1-t)}, \tag{33}$$

$$\lambda(t) = \frac{\sqrt{-\Sigma(t)'}}{2} = \frac{\Sigma_0\sqrt{\Sigma_\epsilon}}{2[\Sigma_0 t + \Sigma_\epsilon(1-t)]}, \tag{34}$$

$$\bar{\lambda}(t) = \sqrt{-\Sigma'(t)} = \frac{\Sigma_0\sqrt{\Sigma_\epsilon}}{\Sigma_0 t + \Sigma_\epsilon(1-t)}, \tag{35}$$

where $\Sigma(t)$ is specified as

$$\Sigma(t) = \frac{\Sigma_0\Sigma_\epsilon(1-t)}{\Sigma_0 t + \Sigma_\epsilon(1-t)}, \tag{36}$$

In equilibrium, the expected profit of each informed investor π_D is

$$\pi_D = \frac{1}{2}\int_0^1 \lambda(t)dt = \frac{\Sigma_0\sqrt{\Sigma_\epsilon}[\ln(\Sigma_0) - \ln(\Sigma_\epsilon)]}{4(\Sigma_0 - \Sigma_\epsilon)}. \tag{37}$$

Investors' trading intensity β is proportional to $1/\sqrt{\Sigma_\epsilon}$. This is sensible as investors will trade more cautiously as they have noisier signals.[5] Surprisingly, λ is finite through the trading period and remains constant as long as $\Sigma_0 = \Sigma_\epsilon$. This is in sharp contrast to the result in BCW (2000) who show that λ goes to infinity near the end of trading in the absence of disclosure.

3 Comparative Dynamics

3.1 Dynamic Trading Patterns, Market Efficiency, and Market Liquidity

In this section, we use the closed-form solution derived in the previous section to study the dynamics and comparative statics of trading, market efficiency, and market liquidity.

In most strategic trading models, the trading volume coming from informed investors

[5]Note that $\Sigma_\epsilon = 2(1-\theta)\sigma_s^2$ and σ_s^2 is the measure of noisiness of investors' signals.

is negligible compared to liquidity traders.[6] However, when disclosure is required, informed investors' trades contain a component that is comparable to that of liquidity traders and informed investors contribute half of the trading volume in the market with disclosure.[7] To mix with liquidity traders, the endogenous random trades of informed investors equal in distribution to that of liquidity traders.

We next examine the comparative statics of $\Sigma(t)$, $\beta(t)$, $\lambda(t)$ with respect to time and the degree of noise in informed investors' signals, as measured by σ_s.

PROPOSITION 1 *The variables $\Sigma(t)^{-1}$, $\beta(t)$ both increase with t and decrease with σ_s. The market depth $1/\lambda(t)$ increases (decreases) over time when $\Sigma_\epsilon < \Sigma_0$ ($\Sigma_\epsilon \geq \Sigma_0$).*

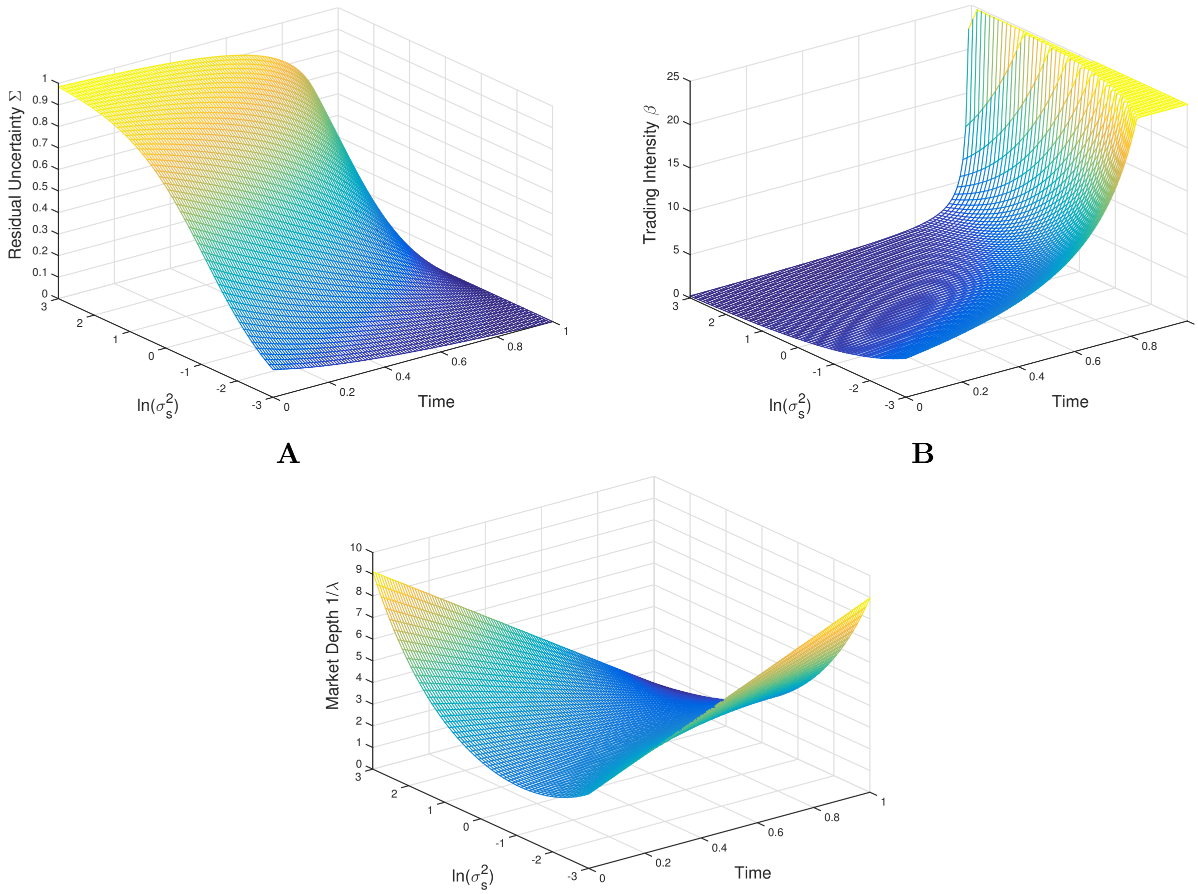
In Figure 1**A** and Figure 1**B**, we plot $\Sigma(t)$ and $\beta(t)$ as functions of t, $\ln(\sigma_s^2)$. As more information is revealed through trading and disclosure, clearly $\Sigma(t)$ will decrease over time. Similarly, as investors learn more from trading and disclosure and market becoming more efficient, the trading intensity increases over time as well. When σ_s is small, informed investors trade very aggressively with each other and thus $\Sigma(t)$ is low and $\beta(t)$ is high. As shown in Figure 1**A**, market becomes less efficient as σ_s increases. While in BCW (2000), the decrease of conditional variance is very steep and goes to infinity near the end of trading, in our model with disclosure, the conditional variance always decreases smoothly. In Figure 1**B**, it is clear that $\beta(t)$ decreases with σ_s. Coarser information makes investors compete with each other less intensively.

The comparative statics of $\lambda(t)$ is more complicated. Following Kyle (1985), we use market depth, $1/\lambda(t)$, to measure market liquidity. When σ_s is high, investors will trade very cautiously initially and only increase their trades aggressively later on. This means that the market depth decreases over time. Figure 1**C** plots market depth as a function of t, $\ln(\sigma_s^2)$. With high (low) σ_s, market depth decreases (increases) over time. When $\rho_0 = 0$, the market depth is a constant.

Disclosure not only increases market efficiency, it also affects how informed investors compete with each other. It is interesting to compare the trading strategy of informed

[6]There are few exceptions in which trading volume coming from informed investors is comparable to liquidity traders. For example, informed investors in Martinez and Rosu (2013)'s model are ambiguity averse and ambiguity aversion induces informed investors to aggressively trade on their signals, so that their optimal trading strategy features a volatility component. In Foucault, Hombert, and Rosu (2012)'s model, a fast informed investor trades on his forecast of short-run price movements before the market maker reacts to news and hence informed order contains a volatility component.

[7]This can be seen clearly from $2\sigma(t)^2 = 1$.



C

Figure 1 Figure 1**A**: Residual uncertainty Σ as a function of $\ln(\sigma_s^2)$, t with disclosure. Figure 1**B**: Trading intensity β as a function of $\ln(\sigma_s^2)$, t with disclosure. Figure 1**C**: Market depth $1/\lambda$ as a function of $\ln(\sigma_s^2)$, t with disclosure. $\sigma_v^2 = 1$, $\theta = 0.5$.

investors in the aggregate to that of a monopolist. Define $\delta(t) \equiv (\Sigma(t) - \Omega(t))/\Sigma(t)$ as we do in the discrete-time setting $\delta_m \equiv (\Sigma_m - \Omega_m)/\Sigma_m$ and let $\rho(t)$ denote the conditional correlation of informed investor's private valuation at time t, by the definition of $\delta(t)$, we have $\rho(t) = 2\delta(t) - 1$. We have the following results.

PROPOSITION 2 *When $\rho_0 = 0$, informed investors trade cooperatively like a monopolistic investor in the aggregate. Conditional correlation $\rho(t)$ of investors' private valuation remains zero throughout the trading period.*

When informed investors' signals are uncorrelated initially, each informed investor's conditional precision is twice of that of the market maker. As trading goes on, since each informed investor knows his own random noise trades, the variance of the noise in the other informed investor's trades is also half of the variance of the noise in the market maker's observation. The conditional precision of each informed investor about the risky asset value remains twice of that of the market. As a result, the conditional correlation of informed investors' signals remains zero. Disclosure makes informed investors cooperate with each other. With uncorrelated signals, market efficiency and market liquidity are the same as if there exists a monopolistic informed investor with all the private signals in the market.

PROPOSITION 3 *When $\rho_0 \neq 0$, as $t \to 1$, $\rho(t) \to 0$ and informed investors' private valuations become conditionally uncorrelated and they eventually all behave in the aggregate like a monopolistic informed investor with all the information in the economy. We have*

$$\lim_{t\to 1} \frac{\Sigma(t)}{\Sigma_\epsilon(1-t)} = 1, \quad \frac{\beta(t)}{1/(\sqrt{\Sigma_\epsilon}(1-t))} = 1, \quad \lim_{t\to 1} \frac{1/\lambda(t)}{2/\sqrt{\Sigma_\epsilon}} = 1.$$

With conditionally uncorrelated signals, informed investors will be cooperative from the beginning to the end. With conditionally correlated signals, informed investors learn to become cooperative. As discussed earlier, the increase in conditional precision of informed investors is twice of that of the market maker. As learning accumulates, the ratio of the conditional precision of informed investors and that of the market maker about the asset value converges to two. The conditional correlation between informed investors' signals converges to zero. This is drastically different from the case without disclosure. In the BCW (2000) model without disclosure, near the end of trading, the ratio of the conditional precision of informed investors and that of the market maker about the asset value converges to 1 as the increase in conditional precision goes to infinity. This holds because the noise in the price comes from the liquidity traders and no one has any extra information about the noise trades. Therefore the increase in conditional precision is the same for the market maker and informed investors. As time goes to 1, the increase in conditional precision goes to infinity and the ratio of conditional precision between informed investors and market maker goes to 1. Informed investors have little informational advantage over the market maker, the conditional correlation of informed investors' private valuation goes to -1 and $1/\lambda(t)$ goes to 0. On the contrary, in continuous-time trading with disclosure, investors learn to become cooperative. The conditional correlation of investors' private valuation converges to zero and $1/\lambda(t)$ goes to a constant.

3.2 The Effects of Disclosure

Next we compare the equilibrium with that obtained by BCW (2000) without disclosure. For comparison, $\hat{\Sigma}(t)$, $\hat{\beta}(t)$, $\hat{\lambda}(t)$, $\hat{\delta}(t)$, and $\hat{\pi}_{\mathrm{D}}$ in the BCW (2000) economy without disclosure correspond to the same parameters without hat in the economy with disclosure.

THEOREM 3 *In the continuous-time trading model without public disclosure, we have*

$$\begin{aligned}
\hat{\Sigma}(t) &= \frac{\Sigma_0 \Sigma_\epsilon}{\Sigma_\epsilon - \Sigma_0 \ln(1-t)}, \\
\hat{\beta}(t) &= \frac{1}{\sqrt{\Sigma_\epsilon}\sqrt{1-t}}, \\
\hat{\lambda}(t) &= \hat{\beta}(t)\hat{\Sigma}(t).
\end{aligned}$$

The following corollary describes how disclosure affects $\beta(t)$, $\Sigma(t)$.

PROPOSITION 4 *The market is more efficient and informed investors' information-based trade is more aggressive, that is*

$$\begin{aligned}
\frac{\Sigma(t)}{\hat{\Sigma}(t)} &= \frac{\Sigma_\epsilon - \Sigma_0 \ln(1-t)}{\Sigma_\epsilon + \Sigma_0 t/(1-t)} < 1, \\
\frac{\beta(t)}{\hat{\beta}(t)} &= \frac{1}{\sqrt{1-t}} > 1,
\end{aligned}$$

Moreover as time approaches 1, we have,

$$\lim_{t\to 1}\frac{\Sigma(t)}{\hat{\Sigma}(t)}=0,\lim_{t\to 1}\frac{\beta(t)}{\hat{\beta}(t)}=\infty,.$$

Disclosure makes the market more efficient. Since informed investors' information-based trade is mixed with random-noise trades, they trade more aggressively with respect to their signals. This effect is most profound near the end of trading as the ratio of residual uncertainty Σ with and without disclosure goes to zero. Figure 2**B** shows the intensity of informed investors' trading in relation to that of informed trading without disclosure. The intensity is greater when disclosure is required. The ratio of trading intensity with and without disclosure is always larger than 1 and goes to infinity near the end of trade.

As a result of more aggressive trading by informed investors and the fact that the random order from all informed investors collectively equals, in distribution, to that of liquidity traders, market becomes more efficient under the disclosure rule. This is clearly demonstrated in Figure 2**A**.

Next we compare market depth, $1/\lambda(t)$ and expected profits of informed investors in the two equilibria with and without disclosure. The expected profits π_{D} are related to market depth as described in Theorem 2:

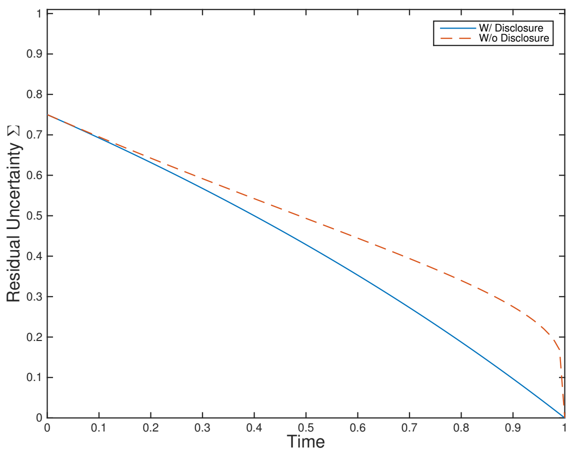
$$\pi_{\mathrm{D}}=\frac{1}{2}\int_0^1\lambda(t)dt.$$

Notice that $\lambda(t)$ represents the expected losses per unit of trade for liquidity traders arriving at time t. The previous relationship holds because the expected profits of informed investors equal to the expected losses of liquidity traders. The following describes the effects of disclosure on these variables.

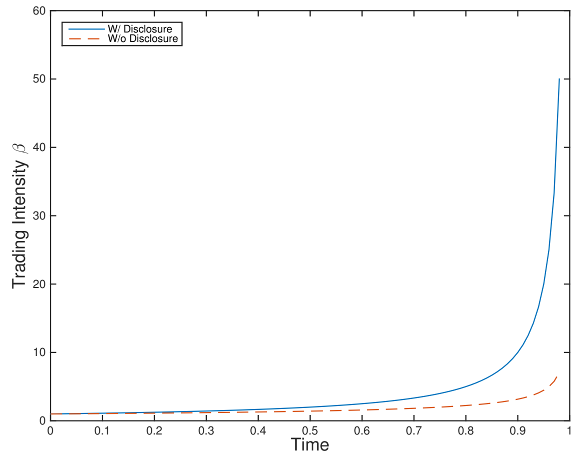
PROPOSITION 5 *As time approaches 1, we have*

$$\lim_{t\to 1}\frac{1/\lambda(t)}{1/\hat{\lambda}(t)}=\infty,$$

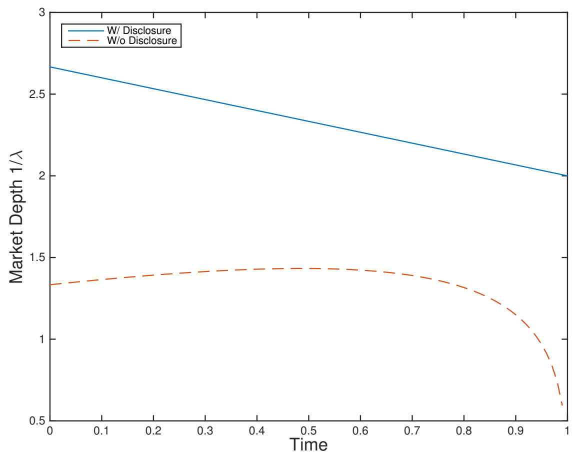
Moreover, when $\rho_0\geq 0$, then $1/\lambda(t)>1/\hat{\lambda}(t)$ for $t\in[0,1]$. In addition, $\pi_D<\hat{\pi}_D$.



A



B



C

Figure 2 Figure 2**A**: Residual uncertainty Σ as a function of time. Figure 2**B**: Trading intensity β as a function of time. Figure 2**C**: Market depth $1/\lambda(t)$ as a function of time. The solid line is for the case with disclosure and the dashed line is for the case without disclosure. Here, $\sigma_v^2 = 1$, $\sigma_s^2 = 1$, $\theta = 0.5$.

We can rewrite the ratio of market depth into the product of three components:

$$\frac{1/\lambda(t)}{1/\hat{\lambda}(t)} = \frac{2}{1} \times \frac{\hat{\beta}(t)}{\beta(t)} \times \frac{\hat{\Sigma}(t)}{\Sigma(t)},$$

Disclosure affects market liquidity in three ways. The first is the randomization effect which will increase market liquidity under disclosure. Other things being equal, this effect will double market liquidity. The second is the higher trading intensity under disclosure which decreases market liquidity. The third is the market efficiency effect which increases market liquidity under disclosure because of a lower residual uncertainty.

Figure 2**C** plots the market depth with positively correlated signals. When $\rho_0 \geq 0$ the last two effects roughly offset each other except near the beginning of trade. The first effect is dominant in the early part of the trading period and market liquidity roughly doubles. In the latter part of the trading period, disclosure makes the market more efficient and the third effect is dominant which causes a higher market liquidity. Therefore, market is always more liquid with disclosure.

In brief, when the noise in informed investors' signals is small, informed investors do not learn from each other as much. As they trade more aggressively on their perceived differences from market expectation under disclosure, market depth is higher with disclosure due to randomization and higher market efficiency. It is interesting to observe that market depth changes over time in a pattern that is different from the no-disclosure case. Without disclosure, market depth first rises and then declines to 0 with positively correlated signals but market depth always rises with negatively correlated signals.

4 Learning from Your Competitor and Herding in Information Acquisition

When σ_s is small, informed investors have precise and substitutive signals and they compete intensively in the presence of disclosure. When σ_s is large, the informed investors each has a very noisy signal, they will learn a lot from each other and the effects of disclosure are more subtle. On the one hand, disclosure makes investors trade more intensively. On the other hand, it also allows informed investors to learn more from each other and thus they become relatively more informed. Therefore the results obtained with small σ_s may not carry through to the case with large σ_s.

PROPOSITION 6 *For $t > 3/4$, there exists σ_s^*, such that for $\sigma_s > \sigma_s^*$, that $1/\lambda(t) < 1/\hat{\lambda}(t)$.*

This is a rather surprising result. Intuitively, one would have expected that disclosure should always increase market liquidity. As discussed earlier, the effect of trade disclosure on market liquidity can be decomposed to three components: randomization effect, learning effect and the market efficiency effect. When σ_s is very large, each informed investor on his own knows very little about the liquidation value of the risky asset. Therefore they learn a lot from the disclosure of informed investors' trades. Since the variance of noise per unit of time in disclosed trades is $2\sigma^2$ for the market maker and σ^2 for each informed investor, informed investors learn faster from disclosed trades than the market maker. When σ_s is very large, learning from public disclosure becomes very significant and this effect dominates the other two effects, which causes the market liquidity to be higher for some t.

The reduction in market liquidity means that informed investors make more profits in some trading periods with disclosure. A natural question is whether disclosure can increase expected profits of informed investors during the whole trading period, which we find possible when σ_s is large.

PROPOSITION 7 *There exists σ_s^{**}, such that for $\sigma_s > \sigma_s^{**}$, $\pi_D > \hat{\pi}_D$.*

The effect of disclosure on informed investors' profits is ambiguous. Other things being equal, disclosure causes informed investors to lose half of their information-based trading profits due to randomization. This results in a reduction of informed investors' profits when σ_s is small. With large σ_s, the results can be reversed. In the latter case, informed investors learn a lot from the disclosed trades about the asset value as they each have very imprecise signals in the beginning. In addition, informed investors learn more from the disclosed trades than the market maker. The increase of precision is twice that of the market maker. Consequently, the benefit of learning by informed investors could more than offset the loss due to randomization and make them earn more profits than what they would receive in a setting without disclosure.

Alternatively, we can view disclosure as an apparatus for coordination. Notice that informed investors' profits would be maximized if they could coordinate and trade at the same intensity as a monopolist with all the signals. When each informed investor has very imprecise signals, they trade very cautiously, far from the level of a monopolist in the case without disclosure. Disclosure of trades releases information and makes them trade more aggressively toward the level of a monopolist. Indeed as shown in Proposition 3, informed investors learn to become cooperative. The increase of trading intensity effectively coordinates their trading activity toward higher profits, and can offset the losses due to randomization when σ_s is small.

Disclosure makes informed investors learn from each other and cooperate. In the case of multiple stocks, this can create herding in information acquisition. Consider an economy with two risky assets and a risky free asset. Let v_j, v_k denote the two risky asset payoffs. For each risky asset, there is a public signal

$$q_l = v_l - \sum_{i=1} s_l^i,\ l = j, k \tag{38}$$

To simplify the analysis, we assume that the signals and payoffs are orthogonal across the two risky asset payoffs. In addition, the variance and correlation of signals are identical for the two risky assets. An informed investors due to information processing capacity can only acquire information on one of the risky assets. We are interested in conditions in which informed investors would be better of acquiring information in the same risky asset. Let M denote the economy in which informed investors will be investigating different risk assets and D denote the economy in which both informed investors will be investigating the same risky asset. Let π_{E} ($\hat{\pi}_{\mathrm{E}}$) denote the expected profits of an informed investor in economy $E = M, D$ in the presence (absence) of disclosure requirement. We first compare the profits with and without competition.

PROPOSITION 8 *There exists $\bar{\sigma}_s$ such that for $\sigma_s > \bar{\sigma}_s$, $\pi_D > \hat{\pi}_M > \pi_M$. However, in the economy without disclosure, we always have $\hat{\pi}_M > \hat{\pi}_D$.*

Competition always reduces an informed investor's profit in the case without disclosure as informed investors learn at the same speed as the market maker. However, in the presence of disclosure, with very large σ_s, informed investors have very noisy signals and are eager to learn from each other. Disclosure of trades facilitates learning among informed investors about the risky asset value at a speed (as measured by the increase in conditional precision) twice as fast as that of the market maker. With very large σ_s, the benefit of learning can offset the loss due to competition and informed investors are better off with competition. Interestingly, learning from each other is so beneficial that an informed investor with disclosure and competition is better off than what he expects to receive with neither disclosure nor competition.

So far we have presented our model assuming informed investors already received signals. What is the effect of disclosure on information acquisition? Our analysis indicates that learning can create synergies in information acquisition in the presence of disclosure. Suppose that each informed investor has decide which stock to acquire information, then disclosure of trades will make investors to acquire information on the stake risky asset when the signals are very noisy.

COROLLARY 3 *In the absence of trade disclosure, investors will always acquire information on different stocks. In the presence of trade disclosure, there exists a $\bar{\sigma}_s$ such that when $\sigma_s > \bar{\sigma}_s$, investors will acquire information on the same stock and when $\sigma_s \leq \bar{\sigma}_s$ investors will acquire information on different stocks.*

Herding in information acquisition happens because informed investors can learn more from each other than what the market can learn from informed investors.

5 Extension

Our model can be extended to an arbitrary number of informed investors with the following modification. Assuming that at time zero a public signal q is announced and

$$q = v - \sum_{i=1}^{N} s^i \tag{39}$$

The correlation of s^i is θ. Each informed investor $i = 1, \ldots, N$ observes the noise in the public signal and thus are able to interpret the public signal better than the market maker. Let

$$\Sigma_0 = \mathrm{var}[v|q] = \frac{N[1 + (N-1)\theta]\sigma_s^2\sigma_v^2}{\sigma_v^2 + N[1 + (N-1)\theta]\sigma_s^2}$$

let

$$\epsilon^i = Ns^i - \sum_{j=1}^{N} s^j$$

and we have

$$\Sigma_\epsilon \equiv \mathrm{var}[\epsilon^1] = N(N-1)(1-\theta)\sigma_s^2$$

Given the public information, the conditional correlation is

$$\rho_0 = \frac{(N-1)\Sigma_0 - \Sigma_\epsilon}{(N-1)(\Sigma_0 + \Sigma_\epsilon)}$$

As before, we introduce the following notation

$$\delta_0 \equiv \frac{\text{var}[v|q] - \text{var}[v|s^1, q]}{\text{var}[v|q]} = \frac{\text{var}^{-1}[v|s^1, q] - \text{var}^{-1}[v|q]}{\text{var}^{-1}[v|s^1, q]} = \frac{1}{N} + \frac{(N-1)\rho_0}{N}. \tag{40}$$

This is a measure of the quality of private information of informed investor 1 and by the argument of symmetry, other informed investors as well. Given these notations, we present the discrete-time model and continuous-time model below:

THEOREM 4 *The necessary and sufficient conditions for a recursive linear symmetric equilibrium to exist are described below. For all $m = 1, \ldots, M-1$ and for all informed investors $i = 1, \ldots, N$,*

$$x_m^i = \frac{\beta_m \Delta t}{N\delta_{m-1}}(V_{m-1}^i - V_{m-1}) + z_m^i \tag{41}$$

$$P_m = V_{m-1} + \lambda_m \left(z_m^0 + \sum_{i=1}^{N} x_m^i \right) \tag{42}$$

$$V_m = V_{m-1} + \bar{\lambda}_m \sum_{i=1}^{N} x_m^i \tag{43}$$

$$\sigma_m^2 = \beta_m \Sigma_m / (N\bar{\lambda}_m) \tag{44}$$

$$\lambda_m = \beta_m \Sigma_{m-1} / (\beta_m^2 \Delta t \Sigma_{m-1} + 1 + N\sigma_m^2) \tag{45}$$

$$V_m^i - V_{m-1}^i = \frac{\Omega_{m-1} - \Omega_m}{\Omega_{m-1}} \left(v - V_{m-1}^i + \sum_{j \neq i} \frac{z_m^j}{\beta_m \Delta t} \right) \tag{46}$$

$$V_m - V_{m-1} = \frac{\Sigma_{m-1} - \Sigma_m}{\Sigma_{m-1}} \left(v - V_{m-1} + \sum_{1 \leq j \leq N} \frac{z_m^j}{\beta_m \Delta t} \right) \tag{47}$$

$$\Omega_m^{-1} = \Omega_{m-1}^{-1} + \beta_m^2 \Delta t / ((N-1)\sigma_m^2) \tag{48}$$

$$\Sigma_m^{-1} = \Sigma_{m-1}^{-1} + \beta_m^2 \Delta t / (N\sigma_m^2) \tag{49}$$

$$E[\pi_m^i | F_{m-1}^i] = \alpha_{m-1}(V_{m-1}^i - V_{m-1})^2 + \zeta_{m-1} \tag{50}$$

$$\lambda_m = \alpha_m \bar{\lambda}_m^2 \tag{51}$$

$$\bar{\lambda}_m = \frac{2\lambda_m}{1 + \lambda_m \beta_m \Delta t (1 - 1/(N\delta_{m-1}))} \tag{52}$$

$$\alpha_{m-1} = \alpha_m \left(1 - \frac{\beta_m^2 \Delta t \Sigma_m}{N\sigma_m^2}\left(1 - \frac{1}{N\delta_{m-1}}\right)\right)^2 \tag{53}$$

$$\zeta_{m-1} = \zeta_m + \alpha_m \beta_m^2 \Delta t \left(\frac{\Omega_m}{(N-1)\sigma_m^2} - \frac{\Sigma_m}{N\sigma_m^2}\right)^2 \left(\Omega_{m-1}\beta_m^2 \Delta t + (N-1)\sigma_m^2\right) \tag{54}$$

subjecting to the boundary conditions

$$\beta_M = \sqrt{\frac{N\delta_{M-1}}{\Sigma_{M-1}\Delta t}}, \tag{55}$$

$$\lambda_M = \frac{\sqrt{N\delta_{M-1}\Sigma_{M-1}/\Delta t}}{1 + N\delta_{M-1}}, \tag{56}$$

$$\alpha_{M-1} = \frac{1}{\lambda_M (1 + N\delta_{M-1})^2}, \tag{57}$$

$$\zeta_{M-1} = 0, \tag{58}$$

and the second-order condition

$$\lambda_m > 0. \tag{59}$$

Similar to the case of two informed investors, the equilibrium can be solved recursively. When Δt goes to zero, the system converges to a set of differential equations which leads to Theorem 5.

THEOREM 5 *As the trading frequency goes to infinity, we obtain closed-form solutions as follows:*

$$\beta(t) = \frac{\sqrt{-\Sigma'(t)}}{\Sigma(t)}, \quad \lambda(t) = \frac{\sqrt{-\Sigma'(t)}}{2}, \quad \bar{\lambda}(t) = \sqrt{-\Sigma'(t)},$$

where

$$\Sigma(t) = \begin{cases} \Sigma_0(1-t) & \text{for } \Sigma_\epsilon = (N-1)\Sigma_0 \text{ or } N = 1, \\ \frac{\Sigma_\epsilon \Sigma_0}{(N-1)\Sigma_0 - \Sigma_\epsilon}\left[((1-B)\,t + B)^{\frac{N}{4-3N}} - 1\right] & \text{otherwise.} \end{cases}$$

with $B = \left(\frac{\Sigma_\epsilon}{(N-1)\Sigma_0}\right)^{3-\frac{4}{N}}$ *. In equilibrium, the expected profit of each informed investor is*

$$\pi_N = \begin{cases} \frac{\sqrt{\Sigma_0}}{2N} & \text{for } \rho_0 = 0 \text{ or } N = 1, \\ \sqrt{\frac{(3N-4)\Sigma_0\Sigma_\epsilon}{N(1-B)((N-1)\Sigma_0-\Sigma_\epsilon)}} \frac{\left|1-B^{\frac{N-2}{3N-4}}\right|}{2|N-2|} & \text{otherwise.} \end{cases} \tag{60}$$

For the purpose of comparison, we restate the BCW (2000) result of continuous trading equilibrium without disclosure in the next theorem.

THEOREM 6 *In the economy with* N *informed investors without disclosure, consider the constant*

$$k = \int_1^\infty x^{\frac{2(N-2)}{N}} e^{\frac{-2\Sigma_\epsilon x}{N\Sigma_0}}\, dx. \tag{61}$$

For each $t < 1$*, define* $\hat{\Sigma}(t)$ *by*

$$\int_1^{\hat{\Sigma}(0)/\hat{\Sigma}(t)} x^{\frac{2(N-2)}{N}} e^{\frac{-2\Sigma_\epsilon x}{N\Sigma_0}}\, dx = kt. \tag{62}$$

with $\hat{\Sigma}(0) = \Sigma_0$*. We have*

$$\hat{\beta}(t) = \left(\frac{k}{\Sigma_0}\right)^{1/2} \left(\frac{\hat{\Sigma}(t)}{\Sigma_0}\right)^{(N-2)/N} \exp\left\{\frac{\Sigma_\epsilon}{N\hat{\Sigma}(t)}\right\}, \tag{63}$$

$$\hat{\lambda}(t) = \hat{\beta}(t)\hat{\Sigma}(t). \tag{64}$$

With respect to the comparative statics of the case with an arbitrary number of informed investors, we have the following results:

PROPOSITION 9 *(i) For* $N = 1$*, we have* $\beta(t) = \hat{\beta}(t)$*,* $\Sigma(t) = \hat{\Sigma}(t)$*,* $\lambda(t) = \hat{\lambda}(t)/2 = 1/(2\sqrt{\Sigma_0})$*;*
(ii) For $N > 1$*, we have:* $\lim_{t\to 1} \frac{\beta(t)}{\hat{\beta}(t)} = \infty$*,* $\lim_{t\to 1} \frac{\Sigma(t)}{\hat{\Sigma}(t)} = 0$*, and*
(iii) $\lim_{t\to 1} \frac{1/\lambda(t)}{1/\hat{\lambda}(t)} = \infty$*;*
(iv)The conditional variance of the asset value $\Sigma(t)$ *decreases with* t *and increases with* σ_s*. The market depth* $1/\lambda(t)$ *increases (decreases) over time when* $\rho_0 > 0$ *(*$\rho_0 \le 0$*);*
(v)When $\rho_0 = 0$*, informed investors trade in aggregate like a monopolistic investor. Therefore, market efficiency and market liquidity are the same as if there exists a monopolistic*

informed investor with all the private signals in the market. Conditional correlation of investors' private valuations remains zero throughout the trading period;
(vi) When $\rho_0 \neq 0$, as $t \to 1$, $\rho(t) \to 0$, informed investors' private valuations become uncorrelated near the end of trading. They learn to cooperate and behave in aggregate like a monopolistic informed investor with all the information in the economy. We have

$$\lim_{t\to 1} \frac{\beta(t)}{1/(\sqrt{S_0}(1-t))} = 1, \quad \lim_{t\to 1} \frac{\Sigma(t)}{S_0(1-t)} = 1, \quad \lim_{t\to 1} \frac{\lambda(t)}{\sqrt{S_0}/2} = 1.$$

Here, $S_0 = \frac{(1-\rho_0)(1-B)\Sigma_0}{\rho_0(3N-4)}$, B is defined in Theorem 5.

Notice that our results on comparative statics obtained with two informed investors broadly hold for larger N. Informed investors also contribute half of the trading volume in the market with disclosure. The conditional variance increases as investors receive noisier signals. Initial market depth is higher with noisier signals as investors trade cautiously initially. However, market depth in the end of trading will be lower with noisier signals as there will be more residual asymmetric information near the end. As a result, market depth will be decreasing with noisy signals and increasing with precise signals. Figure 3**A** plots informed investors' expected profits as a function of $\ln(\sigma_s^2)$. if informed investors have conditionally uncorrelated signals, they coordinate and trade like a monopolist. Moreover, the conditional correlation goes to zero near the end of trading even when investors initially have correlated signals. Informed investors learn to be cooperative. Our numerical analysis shows that disclosure increases the intensity of informed trading and improves market efficiency and this result can be proven for t close to 1. The increase in market efficiency due to disclosure also makes the market depth higher near the end of trading. Our numerical analysis also shows that when investors' signals are positively correlated, disclosure always increases market liquidity.

Next we consider whether informed investors can be better off in the presence of more informed investors due to enhanced learning among informed investors. Let π_N denote what an informed investor would expect to receive in a setting with N informed investors. Let $\pi_{N\to J}$ denote the profits each informed investor would obtain if $N-J$ informed investors leave the market and the other J informed investors will stay and trade in this market.

PROPOSITION 10 *For any $N > 1$, there exists $\dot{\sigma}_s$ such that $\pi_N > \pi_{N\to N-1}$ for all $\sigma_s > \dot{\sigma}_s$. In addition, for $1 < N < 5$, there exists $\bar{\bar{\sigma}}_s$ such that for $\sigma_\epsilon > \bar{\bar{\sigma}}_s$, $\pi_N > \pi_M$.*

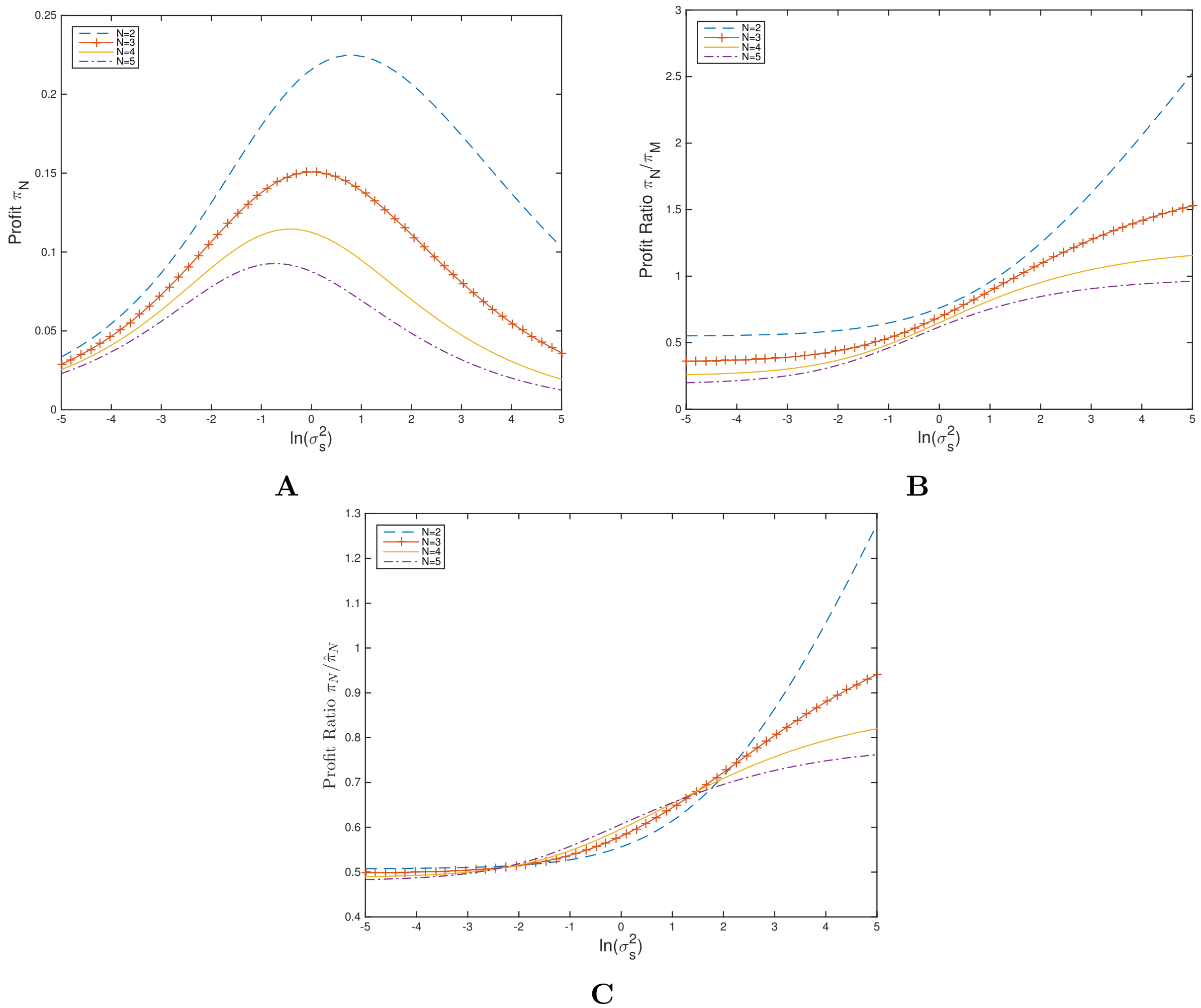


Figure 3 Figure 3**A**: Informed investors' expected profit π_N as a function of $\ln(\sigma_s^2)$ for $N = 2, 3, 4, 5$ with disclosure. Figure 3**B**: The ratio of informed investors' total profits π_N with many competitive informed investors and π_{M} with a monopolistic investor as a function of $\ln(\sigma_s^2)$ for $N = 2, 3, 4, 5$. Figure 3**C**: The ratio of informed investors' total profits with disclosure (π_N) and without disclosure ($\hat{\pi}_N$) as a function of $\ln(\sigma_s^2)$ for $N = 2, 3, 4, 5$.

However, in the economy without disclosure, a monopolistic informed investor is always worse off in the presence of competition, i.e., we always have $\hat{\pi}_M > \hat{\pi}_N$, for all $N > 1$.

Just like the case with two informed investors, $N-1$ informed investors can benefit from the participation of one more informed investor, if they collectively learn a lot from the new participant through trading. Indeed, learning can be so beneficial that a monopolist will be better off if $N-1$ informed investors all participate when $N < 5$. However, as N goes to infinity, each informed investor's profit goes to zero. In Figure 3**B**, we show numerically that for $N = 5$, a monopolist would prefer the other four informed investors not to participate in the market.[8]

With two informed investors, it is possible that disclosure increases the aggregate profits of informed investors. We show numerically in Figure 3**C** that this is impossible when $N > 2$. With larger N, each informed investor will learn at the speed $N/(N-1)$ times that of the market maker. However $N/(N-1)$ is decreasing in N, therefore, for larger N the benefit of learning and coordination is not big enough to offset the loss due to randomization.

Learning among informed investors implies that there could exist herding in information acquisition. Suppose there are N risky assets with independent and identically distributed signals and payoffs across the risky assets. Let v_j, $j = 1, \ldots, N$ denote the risky asset payoffs and

$$q_j = v_j - \sum_{j=1} s_j^i, j = 1, \ldots, N \tag{65}$$

are announced at time 0. Then there could exist herding in information acquisition.

COROLLARY 4 *Suppose $1 < N < 5$. If $\sigma_s > \bar{\bar{\sigma}}_s$, there exist a herding equilibrium in which all informed investors acquire information on the same stock.*

With noisy and complementary information, investors learn from each other and they herd on to stocks on which there are other informed investors trading. For larger N, competition will be too strong for investors to acquire information on the same stock. However, they could exist partial herding in that informed investors will cluster on a few risky assets rather than being the lone informed investor in a single risky asset.

[8] This holds also for $N > 5$ numerically although we cannot provide an analytical proof for this result.

6 Conclusion

What are the effects of public disclosure of trading? In a setting with two informed investors, we show that informed investors will randomize their trades to hide their private information and to manipulate market maker's and others' beliefs. As a result, they sometimes trade against their own valuation. The instantaneous variance of informed investors' trade is the same as that of liquidity traders. Similar to the single informed investor model of HHL (2001), the market is more efficient with trade disclosure.

With more than one informed investor in the market, disclosure facilitates learning among informed investors. Contrary to the model of BCW (2000) in which informed investors learn at the same speed (measured by the increase of conditional precision) as the market maker, in our model informed investors learn twice as fast as the market maker because they know the random component in their own trades. The ratio of conditional precision of informed investors to that of the market maker converges to two from above (below) when investors have positively (negatively) correlated signals. Near the end of trading, investors' signals become conditionally uncorrelated and they trade in aggregate like a monopolist. Learning makes informed investors cooperate. If investors started with uncorrelated signals, they behave in aggregate like a monopolist throughout trading.

With very noisy signals, learning becomes so important that informed investors make more expected profits in the presence of disclosure. In addition, an informed investor could learn so much from disclosure that he makes more profits with competition than trading alone. Synergy due to mutual learning also implies that informed investors may herd in information acquisition. In the case of multiple risky assets, informed investors will prefer to acquire information on the same risky asset when they have very noisy signals.

Disclosure also changes the inter-temporal patterns of the market liquidity. In the model of BCW (2000) without disclosure, informed investors' conditional precision about asset value over that of the market maker converges to 1. Therefore conditional correlation goes to -1 and informed investors will eventually be on the other side of the market and market liquidity goes to zero as they cluster their trades near the end of trading. With precise signals, market liquidity will first increase and then decrease. With noisy signals, market liquidity always decreases over time. On the contrary, in our model, market liquidity is always finite. When informed investors have very noisy signals they will trade more cautiously in the beginning. As time goes on, investors learn more and trade more aggressively, and market liquidity will decrease over time. With small noise in informed investors' private signals, informed investors will trade aggressively initially, which results in a lower market liquidity that increases over time.

In the extension to three or more informed investors, each informed investor still learns more than the market maker. However the speed of learning measured by the derivative of conditional precision is $N/(N-1)$ of that of the market maker. Thus the relative advantage of learning through disclosure for informed investors over the market maker is decreasing with N. We show that for noisy signals, the first $N-1$ informed investors are better off if the Nth informed investor is present in the market. The reduction in the relative speed of learning causes the gains informed investors receive from learning to be lower with higher N. Nevertheless, when $N < 5$, a monopolistic informed investor still prefers the presence of all remaining $N-1$ informed investors in the trading game when signals are very noisy, which will never happen in BCW (2000). However, for $N > 2$, disclosure always makes informed investors worse off. For larger N, potential gains through learning from each other is lower and is not enough to offset the losses due to random noise trades. For $1 < N < 5$, when there are multiple risky assets, investors will herd to acquire information on the same risky asset when they . When information acquisition is endogenized, due to mutual learning among informed investors, there could exist herding equilibria in which an investor will become informed only if he believes all others will become informed.

We considered only the case in which the signals have a symmetric structure. That is they all have the same correlation with each other and the same variance. In the future, it would be interesting to relax this restriction and it is possible that some informed investors benefit from disclosure while others would be worse off. Similarly, with asymmetric information structure, it is also possible that some informed investors may prefer more informed investors to learn from each other while others would be better off with less competition.

Our model provides the first example in which informed investors are better with more public information. It is worthwhile to examine if this also holds in cases of information disclosure of signals about asset value, which we leave for future research.

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Appendices

A Proofs for Section 2

Proof of Theorem 1 We focus on proving the necessity of the claimed equations. The sufficiency of these equations can be established by reversing the necessity arguments (see the end of this proof for more details). So in the rest of this proof except in the last paragraph, we assume that a symmetric linear equilibrium exists, and we prove the claimed equations.

We first prove equations (11) to (14) simply by assuming that each informed investor follows Strategy $\star$. These equations will be used in the inductive proofs for other equations.

First, we can easily check the correctness of equations (11) and (12) by the fact that the expectation of a normal variable is the precision-weighted average of all received signals. Moreover, the updating rule of normally distributed variables states that posterior precision equals prior precision plus the precision of the noise of the signals. Hence, we immediately establish the correctness of equations (13) and (14).

Before proving the rest of the desired equations, we first establish the following useful lemma.

LEMMA 1 *Assume (1) each informed investor believes that all other informed investors follow Strategy $\star$, and (2) the market maker believes that all informed investors follow Strategy $\star$. Then,*

$$\sum_{1 \le i \le 2} (V_m^i - V_m) = 2\delta_m (v - V_m).$$

Proof First, it is easy to check the correctness of the following mathematical identity by properties of normal variables

$$\Omega_0 = \frac{1}{2}(1 - \rho_0)\Sigma_0. \tag{A1}$$

Using this relation and equations (13) and (14), we can easily check

$$\frac{\Omega_m}{\Omega_0}\rho_0 + 1 = 2\delta_m. \tag{A2}$$

In what follows, define

$$U_m^i \equiv E[v - s^i - \frac{1}{2}q | F_m^i]$$

where the expectation is computed after trade disclosures in period m. Equivalently, we could have defined $U_m^i \equiv V_m^i - s^i - \frac{1}{2}q$.

Since the expected value of a normal variable is equal to the precision-weighted average of all received signals, we have

$$\begin{aligned} U_m^j &= \frac{\Omega_m}{\Omega_0} U_0^j + \Omega_m \sum_{1 \le k \le m} \left[\left(\frac{1}{\Omega_k} - \frac{1}{\Omega_{k-1}} \right) \left(s^i + \frac{z_k^i}{\beta_k \Delta t} \right) \right] \\ &= \frac{\Omega_m}{\Omega_0} U_0^j + \Omega_m \sum_{1 \le k \le m} \left[\frac{\beta_k^2 \Delta t}{\sigma_m^2} \left(s^i + \frac{z_k^i}{\beta_k \Delta t} \right) \right], \end{aligned} \tag{A3}$$

where the second equation follows from equation (13). (It is easy to verify that equation (13) holds when each informed investor merely *believes* all other informed investors follow Strategy $\star$.) Similarly,

$$V_m = \frac{\Sigma_m}{\Sigma_0} V_0 + \Sigma_m \sum_{1 \le k \le m} \left[\frac{\beta_k^2 \Delta t}{2\sigma_m^2} \sum_{1 \le i \le 2} \left(s^i + \frac{z_k^i}{\beta_k \Delta t} \right) \right]. \tag{A4}$$

Summing up equation (A3) over $j = 1, 2$, we have

$$\begin{aligned} \sum_{1 \le j \le 2} U_m^j &= \frac{\Omega_m}{\Omega_0} \left(\sum_{1 \le j \le 2} V_0^j - v \right) + \Omega_m \sum_{1 \le k \le m} \left[\frac{\beta_k^2 \Delta t}{\sigma_m^2} \sum_{1 \le i \le 2} \left(s^i + \frac{z_k^i}{\beta_k \Delta t} \right) \right] \\ &= \frac{\Omega_m}{\Omega_0} \left(\frac{\sigma_v^2 (2q + (1+\theta)(v-q))}{\sigma_v^2 + (1-\theta^2)\sigma_s^2} - v \right) + 2 \frac{\Omega_m}{\Sigma_m} \left(V_m - \frac{\Sigma_m}{\Sigma_0} V_0 \right) \\ &= \frac{\Omega_m}{\Omega_0} \rho_0 v + 2 \frac{\Omega_m}{\Sigma_m} V_m \quad \text{(by equation (A4))} \\ &= (2\delta_m - 1) v + 2 \frac{\Omega_m}{\Sigma_m} V_m \quad \text{(by equation (A2)).} \end{aligned}$$

The last equation is only a slight variation of the equality claimed in the lemma. □

We have thus completed the proof of Lemma 1. Using the results established in proving the lemma, we next prove that Strategy (6) satisfies equation ($\star$). In equation (6), x_m^i consists of a random component (z_m^i), a component based on public information ($\frac{\beta_m \Delta t}{N \delta_{m-1}} V_{m-1}$), and a private-information-related component ($\frac{\beta_m \Delta t}{2\delta_{m-1}} V_{m-1}^i$). By equation (A3), the only private component in $\frac{\beta_m \Delta t}{2\delta_{m-1}} V_{m-1}^i$ is equal to

$$\frac{\beta_m \Delta t}{2\delta_{m-1}} \left(s^i + \frac{\Omega_{m-1}}{\Omega_0} \rho_0 s^i \right) = \beta_m \Delta t s^i \quad \text{(by equation (A2))}.$$

This proves that Strategy (6) satisfies equation ($\star$). Moreover, our arguments also imply that to support a symmetric linear equilibrium, x_m^i must have the following form:

$$x_m^i - z_m^i = \frac{\beta_m \Delta t}{2\delta_{m-1}} V_{m-1}^i + \text{ a public-information-based component.} \tag{A5}$$

Using Lemma 1 and equation (6), we have

$$\sum_{1 \le i \le 2} x_m^i = \beta_m \Delta t \left(v - V_{m-1} + \sum_{1 \le i \le 2} \frac{z_m^i}{\beta_m \Delta t} \right). \tag{A6}$$

Therefore, using equation (12) we immediately obtain equation (8) and equation (9) (the derivation of equation (9) also needs equation (14)). Note that in a symmetric linear equilibrium, the value-updating rules must be of the form specified in equation (8). Our arguments in this paragraph together with equation (A5) also show that to support a symmetric linear equilibrium, equation (6) must hold.

Using equation (A6) and the rules of conditional expectation of normally distributed variables, we immediately obtain equation (7) with

$$\begin{aligned} \lambda_m &= \frac{\operatorname{cov}_{m-1}\left[v, \sum_{1 \le j \le 2} x_m^j + z_m^0\right]}{\operatorname{var}_{m-1}\left[z_m^0 + \sum_{1 \le j \le 2} x_m^j\right]} \\ &= \frac{\beta_m \Sigma_{m-1}}{\beta_m^2 \Delta t \Sigma_{m-1} + 1 + 2\sigma_m^2}. \end{aligned}$$

The last equation is exactly equation (10).

We next proceed to prove equations (15) to (24) by backward induction on m, starting with the last period $m = M$. As there are no more trading opportunities after the last period, the maximization problem for each informed investor i is the same as the case without disclosure that has been derived in Foster and Viswanathan (1996) and Cao (1995). In particular, we know that the expected profit function of informed investor i has the form described in equation (15) with the boundary conditions specified in equations (20) to (24).

Thus, we have completed the base step. Next, we assume equations (15) to (19) are correct for period $m+1$ and prove them for period m. By the induction hypothesis, immediately after the mth period disclosure, the expected profits for future trades (i.e., trades from period $m+1$ onwards) can be written as,

$$E_m^i\left[\pi_{m+1}^i\right] \equiv E\left[\pi_{m+1}^i | F_m^i\right] = \alpha_m (V_m^i - V_m)^2 + \zeta_m.$$

Hence, the maximization problem of informed investor i immediately after the $(m-1)$th period trade disclosure is:

$$\max_{x_m^i} E_{m-1}^i\left[x_m^i\left(v - V_{m-1} - \lambda_m\left(z_m^0 + \sum_{1\le j\le 2} x_m^j\right)\right) + \alpha_m (V_m^i - V_m)^2\right] + \zeta_m \tag{A7}$$

where the two terms inside the squared brackets represent the profit of the mth trade and the total profit of all future trades.

For informed investor i to follow a random strategy, he must be indifferent between different values of x_m^i. Thus, the coefficients of $(x_m^i)^2$ and x_m^i in Expression (A7) must be zero. These two restrictions respectively imply

$$\lambda_m = \alpha_m \bar{\lambda}_m^2, \quad \text{and} \tag{A8}$$

$$E_{m-1}^i\left[v - V_{m-1} - \lambda_m x_m^j\right] = 2\alpha_m \bar{\lambda}_m E_{m-1}^i\left[V_m^i - V_{m-1} - \bar{\lambda}_m x_m^j\right]. \tag{A9}$$

Note that equation (A8) is the same as equation (16). In what follows, we show that equations (A8) and (A9) together imply equation (17). On the other hand, by Lemma 1,

$$x_m^j = \beta_m \Delta t\left(v - V_{m-1} - \frac{1}{2\delta_{m-1}}(V_{m-1}^i - V_{m-1})\right) + z_m^j. \tag{A10}$$

Hence,

$$E_{m-1}^i[x_m^j] = \beta_m \Delta t\left(1 - \frac{1}{2\delta_{m-1}}\right)\left(V_{m-1}^i - V_{m-1}\right)$$

Applying this relation to equation (A9), we obtain

$$\frac{1 - \lambda_m \beta_m \Delta t + \lambda_m \beta_m \Delta t/(2\delta_{m-1})}{1 - \bar{\lambda}_m \beta_m \Delta t + \bar{\lambda}_m \beta_m \Delta t/(2\delta_{m-1})} = 2\alpha_m \bar{\lambda}_m.$$

Now we multiply both sides of the preceding equation with the denominator of the left-hand side of the equation, and then we use equation (A8) to substitute all the $\alpha_m\bar{\lambda}^2$ terms by λ_m. This leads to

$$2\alpha_m\bar{\lambda}_m = 1 + \lambda_m\left(\beta_m\Delta t - \frac{\beta_m\Delta t}{2\delta_{m-1}}\right).$$

Next, multiplying both sides of the above equation with $\bar{\lambda}_m$ and using equation (A8) to substitute $\alpha_m\bar{\lambda}^2$ by λ_m, we immediately obtain equation (17).

Since we have established that informed investor i is indifferent to x_m^i, Expression (A7) can be simplified by setting $x_m^i = 0$. Thus,

$$\begin{aligned} E_{m-1}^i[\pi_m^i] &= \alpha_m E_{m-1}^i\left[\left(V_m^i - V_m\right)^2\right] + \zeta_m \\ &= \alpha_m\left(E_{m-1}^i\left[V_m^i - V_m\right]\right)^2 + \alpha_m \mathrm{var}_{m-1}^i[V_m^i - V_m] + \zeta_m \end{aligned} \tag{A11}$$

On the other hand, since we have assumed $x_m^i = 0$ in the profit calculation, using the updating rule for normal variables we have

$$\begin{aligned} V_m^i &= \frac{\Omega_m}{\Omega_{m-1}}V_{m-1}^i + \frac{\Omega_{m-1} - \Omega_m}{\Omega_{m-1}}\left(v + \frac{z_m^j}{\beta_m\Delta t}\right) \\ &= \frac{\Omega_m}{\Omega_{m-1}}V_{m-1}^i + \frac{\Omega_m\beta_m^2\Delta t}{\sigma_m^2}\left(v + \frac{z_m^j}{\beta_m\Delta t}\right), \end{aligned} \tag{A12}$$

where the second equation follows from equation (13). Moreover, using the pricing rules by market maker and applying equation (A10), we have

$$\begin{aligned} V_m &= V_{m-1} + \bar{\lambda}_m\beta_m\Delta t\left(v - V_{m-1} - \frac{V_{m-1}^i - V_{m-1}}{2\delta_{m-1}}\right) + \bar{\lambda}_m z_m^j \\ &= V_{m-1} + \frac{\beta_m^2\Sigma_m\Delta t}{2\sigma_m^2}\left(v - V_{m-1} - \frac{V_{m-1}^i - V_{m-1}}{2\delta_{m-1}} + \frac{z_m^j}{\beta_m\Delta t}\right), \end{aligned} \tag{A13}$$

where the second equation follows from equation (9).

Now, using equations (A12) and (A13) and the fact that v is independent of z_m^j, we have

$$\mathrm{var}_{m-1}^i\left[V_m^i - V_m\right] = \left(\frac{\Omega_m\beta_m^2\Delta t}{\sigma_m^2} - \frac{\beta_m^2\Sigma_m\Delta t}{2\sigma_m^2}\right)^2\left(\Omega_{m-1} + \frac{\sigma_m^2}{\beta_m^2\Delta t}\right) \tag{A14}$$

Moreover,

$$E_{m-1}^i[V_m^i - V_m] = V_{m-1}^i - E_{m-1}^i[V_m]$$

$$= \left(1 - \frac{\beta_m^2 \Sigma_m \Delta t}{2\sigma_m^2}\left(1 - \frac{1}{2\delta_{m-1}}\right)\right)(V_{m-1}^i - V_{m-1}), \tag{A15}$$

here the last equation follows from equation (A13). Substituting equations (A14) and (A15) into equation (A11), we immediately see that equation (15) is correct for m with α and ζ satisfying equations (19) and (53). This completes our inductive step.

So far, we have proved all the desired equations as necessary conditions to support a symmetric linear equilibrium. In proving these equations, we have used (1) the rationality of the market maker's pricing rules and value-updating rules, and (2) the optimality of all informed investors' trading strategies. Moreover, by reversing these arguments, we can easily check that when these equations indeed hold, (1) the pricing rules and value-updating rules are indeed rational for the market maker, and (2) the trading strategies of all informed investors are indeed optimal. Therefore, all these equations collectively form a set of sufficient conditions to support a symmetric linear equilibrium. □

Numerical methods in solving the system of equations in Theorem 1

The whole recursive system of α_m, β_m, λ_m, $\bar{\lambda}_m$, Σ_m, Ω_m, and ζ_m can be numerically solved by first conjecturing a value of Ω_{M-1} and then solving recursively for $\Omega_{M-2}, \ldots, \Omega_0$. Given the conjectured Ω_{M-1}, we can compute δ_{M-1}, since the definition of δ_M and equations (13) and (14) imply

$$2\delta_{M-1} = 1 + \frac{\Omega_{M-1}}{\Omega_0}(2\delta_0 - 1).$$

From Ω_{M-1} and δ_{M-1}, we can now derive Σ_{M-1}. From the boundary condition in equation (57), we can determine α_{M-1}. Now again we conjecture a value for Ω_{M-2}, which allows us to derive δ_{M-2} and Σ_{M-2} as before. From equations (9) and (14),

$$\Sigma_{M-1}^{-1} = \Sigma_{M-2}^{-1} + \bar{\lambda}_{M-1}\beta_{M-1}\Delta t/\Sigma_{M-1}.$$

Consequently, we obtain $\beta_{M-1}\bar{\lambda}_{M-1}$. Comparing equation (10) and equation (16), we arrive at

$$\beta_{M-1}\Sigma_{M-2}/(\beta_{M-1}^2 \Delta t \Sigma_{M-2} + 1 + 2\sigma_{M-1}^2) = \bar{\lambda}_{M-1}^2 \alpha_{M-1}.$$

In the preceding equation, we can use the derived expression for $\beta_{M-1}\bar{\lambda}_{M-1}$ to substitute $\bar{\lambda}_{M-1}$ for β_{M-1}, and we can use equation (9) to substitute $\bar{\lambda}_{M-1}$ for σ_{M-1}^2. Doing so results in an equation with $\bar{\lambda}_{M-1}$ being the only unknown. Solving the resulting equation gives a formula for $\bar{\lambda}_{M-1}$. Next we can derive β_{M-1} from $(\beta_{M-1}\bar{\lambda}_{M-1})/\bar{\lambda}_{M-1}$, λ_{M-1} from equation (51), and σ_{M-1}^2 from equation (9). Given the expressions for λ_{M-1}, $\bar{\lambda}_{M-1}$, β_{M-1}, and σ_{M-1}^2, we can now check whether equation (17) holds or not. If it doesn't, we modify our initial value of Ω_{M-2} until it holds. We repeat the procedure to derive $\Omega_{M-3}, \ldots, \Omega_0$. If the derived Ω_0 is different from the initial given value, we adjust Ω_{M-1} and repeat the whole procedure until the derived Ω_0 equals to the initial given value.

Proof of Corollary 1 When $\rho_0 = 0$, we have $\Sigma_0 = \Sigma_\epsilon$, $\delta_m = 1/2$ from equation (A2) and $\Sigma_m = 2\Omega_m$ by the definition of δ_m. Plugging $\delta_m = 1/2$ into equation (18) gives $\alpha_m \equiv \alpha$ and combining $\Sigma_m = 2\Omega_m$, equations (19) and (23) gives $\zeta_m \equiv 0$, for all $m = 1, \ldots, M-1$. Plugging $\delta_m = 1/2$ into equation (17) gives $\lambda_m = \bar{\lambda}_m/2$, which along with equation (16) leads to $\lambda_m = 1/(4\alpha)$.

From equations (9) and (14), we have

$$\begin{aligned}
\bar{\lambda}_m &= \frac{\beta_m}{2\sigma_m^2}\left(\frac{1}{\Sigma_{m-1}} + \frac{\beta_m^2\Delta_t}{2\sigma_m^2}\right)^{-1} \\
&= \frac{\beta_m\Sigma_{m-1}}{2\sigma_m^2 + \beta_m^2\Sigma_{m-1}\Delta t} \qquad \text{(A16)}
\end{aligned}$$

which along with $\bar{\lambda}_m = 2\lambda_m$ and equation (10) leads to

$$\bar{\lambda}_m = \beta_m\Sigma_{m-1}, \text{ and} \qquad \text{(A17)}$$

$$2\sigma_m^2 = 1 - \beta_m^2\Sigma_{m-1}\Delta t = 1 - \frac{\bar{\lambda}_m^2\Delta_t}{\Sigma_{m-1}} = 1 - \frac{\Delta t}{4\alpha^2\Sigma_{m-1}}. \qquad \text{(A18)}$$

From equation (14), we have

$$\begin{aligned}
\Sigma_m &= \frac{2\sigma_m^2\Sigma_{m-1}}{\beta_m^2\Sigma_{m-1}\Delta t + 2\sigma_m^2} = \Sigma_{m-1}\left(1 - \frac{\Delta t}{4\alpha^2\Sigma_{m-1}}\right) \\
&= \Sigma_{m-1} - \frac{\Delta t}{4\alpha^2} = \Sigma_0 - \frac{m\Delta t}{4\alpha^2}
\end{aligned}$$

Combining equation (21), equation (22), and $\delta_m = 1/2$ gives

$$\alpha = \frac{1}{2\sqrt{\Sigma_{M-1}/\Delta t}}$$

So we have

$$\Sigma_{M-1} = \frac{\Delta t}{4\alpha^2} = \Sigma_0 - \frac{(M-1)\Delta t}{4\alpha^2}, \text{ and}$$
$$\alpha = \frac{1}{2\sqrt{\Sigma_0}} \qquad \text{(Remember } \Delta t = 1/M\text{)}.$$

β_m and σ_m^2 can be calculated from equations (A17) and (A18), which completes the proof. □

Proof of Corollary 2 From equation (17), as $\Delta t \to 0$ we have $\lambda_m = \bar{\lambda}_m/2$ and equation (28) comes directly from equation (16).

Combining equations (9), (10) and (14) gives

$$\begin{aligned}\frac{\bar{\lambda}_m}{\lambda_m} &= \frac{\beta_m(2\sigma_m^2\Sigma_{m-1})/(2\sigma_m^2 + \beta_m^2\Sigma_{m-1}\Delta t)}{2\sigma_m^2}\frac{1 + 2\sigma_m^2 + \beta_m^2\Sigma_{m-1}\Delta t}{\beta_m\Sigma_{m-1}} \\ &= \frac{1 + 2\sigma_m^2 + \beta_m^2\Sigma_{m-1}\Delta t}{2\sigma_m^2 + \beta_m^2\Sigma_{m-1}\Delta t} = 2\end{aligned}$$

which means $2\sigma_m^2 = 1$ as $\Delta t \to 0$ and hence equations (29) and (30).

Combining equations (9) and (10) and $2\sigma_m^2 = 1$ gives equations (25) and (26). Equations (31) and (32) come directly from equations (18) and (19) and $\sigma_m^2 = 1/2$. □

Proof of Theorem 2

Combining equation (16), equation (21), and equation (22) gives us

$$\frac{\lambda_{M-1}}{\bar{\lambda}_{M-1}^2}(1 + 2\delta_{M-1})\sqrt{2\delta_{M-1}\Sigma_{M-1}} = \sqrt{\Delta t}.$$

From equation (A2), we know $2\delta_{M-1} > \min\{1, 1+\rho_0\} > 0$. As $\Delta t \to 0$, $\lambda_{M-1}/\bar{\lambda}_{M-1} = 1/2$, we must have

$$\Sigma_{M-1} \to 0, \text{ or} \tag{A19a}$$
$$\bar{\lambda}_{M-1} \to +\infty. \tag{A19b}$$

Let $\Gamma(t) = \Sigma^{-1}(t)$, then we have

$$\beta(t) = \sqrt{\frac{d\Gamma(t)}{dt}} = \sqrt{\Gamma'(t)}, \qquad \text{(by equation (30))}$$
$$\bar{\lambda}(t) = \beta(t)\Sigma(t) = \sqrt{\Gamma'(t)}/\Gamma(t). \qquad \text{(by equation (25))}$$

Define $A \equiv \frac{2\rho_0}{(1-\rho_0)\Sigma(0)} = \frac{2\delta_0 - 1}{\Omega(0)} = \frac{\Sigma_0 - \Sigma_\epsilon}{\Sigma_0\Sigma_\epsilon}$, then we have

$$\begin{aligned}
\Omega(t) &= (2/\Sigma(t) + (2\delta_0 - 1)/\Omega(0))^{-1} \qquad \text{(by equations (29) and (30))} \\
&= (A + 2\Gamma(t))^{-1}, \\
1 - \frac{1}{2\delta(t)} &= 1 - \frac{1}{1 + (2\delta_0 - 1)\Omega(t)/\Omega(0)} \qquad \text{(by equation (A2))} \\
&= 1 - \frac{1}{1 + A(A + 2\Gamma(t))^{-1}} \\
&= \frac{A}{2(A + \Gamma(t))}.
\end{aligned}$$

Plugging the above equation into equation (31) gets

$$\alpha'(t) = \beta(t)\left(1 - \frac{1}{2\delta(t)}\right) = \frac{A\sqrt{\Gamma'(t)}}{2(A + \Gamma(t))}$$

And at the same time, we have

$$\begin{aligned}
\alpha'(t) &= \left(\frac{1}{2\beta(t)\Sigma(t)}\right)' = \left(\frac{\Gamma(t)}{2\sqrt{\Gamma'(t)}}\right)' \\
&= \frac{1}{2}\left((\Gamma'(t))^{\frac{1}{2}} - \frac{1}{2}\Gamma(t)(\Gamma'(t))^{-\frac{3}{2}}\Gamma''(t)\right)
\end{aligned}$$

which means $\Gamma(t)$ satisfies the following ordinary differential equation

$$\frac{\Gamma''(t)}{\Gamma'(t)} + \frac{2\Gamma'(t)}{-A - \Gamma(t)} = 0 \tag{A20}$$

with initial condition $\Gamma(0) = \Sigma_(0)^{-1} = \Sigma_0^{-1}$ and terminal condition (A19).

In the case $\rho_0 = 1$, $A = \infty$, and hence the above equation implies[9]

$$0 = \frac{d}{dt}\left[\ln\left(\Gamma'(t)\right)\right].$$

Thus,

$$\Gamma'(t) = C_0 \text{ for some constant } C_0 > 0,$$

which in turns implies

$$\Sigma(t) = \frac{1}{\Gamma(t)} = \left(C_0 t + C_1\right)^{-1} \text{ for some constant } C_1. \tag{A21}$$

But there are no constants $C_0 > 0$ and C_1 which can make the above $\Sigma(t)$ satisfy either $\Sigma(1) = 0$ or $\lim_{t\to 1}\bar{\lambda}(t) = \sqrt{-\Sigma'(1)} = +\infty$, as required by equation (A19). This completes the proof that a linear equilibrium does not exist for $\rho_0 = 1$.

In the rest of the proof, we assume $\rho_0 \neq 1$ that will ensure a finite A in the rest of the proof. Under this assumption, we prove that equation (A20) has a unique solution of $\Sigma(t)$ as described in the theorem.

By equation (A20),

$$0 = \frac{d}{dt}\left[\ln\left(\Gamma'(t)(\Gamma(t) + A)^{-2}\right)\right].$$

Hence,

$$\Gamma'(t)\left(\Gamma(t) + A\right)^{-2} = C_2 \text{ for some constant } C_2,$$

In the case of $\rho_0 = 0$, we have $A = 0$. Hence, the above equation is equivalent to $\Gamma^{-2}(t)\Gamma'(t) = C_2$, which implies that $\Sigma(t) = 1/\Gamma(t)$ is linear in t. Hence, the desired formula for $\Sigma(t)$ follows immediately from the boundary condition $\Sigma(1) = 0$.

For the case of $\rho_0 \neq 0$, we can make a change of variable as $\Gamma(t) = A\frac{R(t)}{1-R(t)}$, the above equation becomes

$$R'(t) = AC_2.$$

From this and the boundary conditions on $R(0)$ and $R(1)$, we obtain

$$\frac{1}{A\Sigma(t) + 1} = \frac{1}{A\Sigma(1) + 1}t + \frac{1}{A\Sigma(0) + 1}(1 - t). \tag{A22}$$

[9] To be completely formal and to avoid dividing by 0, we should have directly derived the desired equation below. But this is a straightforward exercise by using the argument for obtaining equation (A20).

Taking derivatives with respect to t in the above equation, we know that $\Sigma'(1)$ is bounded. Hence, from the proved formula $\bar{\lambda}(t) = \sqrt{-\Sigma'(t)}$, we know that $\lim_{t\to 1} \bar{\lambda}$ is finite. Hence, from equation (A19), we must have $\Sigma(1) = 0$. Plugging $\Sigma(1) = 0$ into equation (A22), we immediately arrive at the claimed formula for $\Sigma(t)$. The expressions for $\beta(t)$, $\lambda(t)$, and $\bar{\lambda}(t)$ are directly followed from the formula of $\Sigma(t)$.

The expected losses per unit of trade from noise traders arrive at time t, $\lambda(t)$, equal to the expected profits of informed investors. Thus, we have

$$\pi_{\mathrm{D}} = \frac{1}{2}\int_0^1 \lambda(t)dt = \frac{1}{2}\int_0^1 \frac{\Sigma_0\sqrt{\Sigma_\epsilon}}{2[\Sigma_0 t + \Sigma_\epsilon(1-t)]}dt = \frac{\Sigma_0\sqrt{\Sigma_\epsilon}[\ln(\Sigma_0) - \ln(\Sigma_\epsilon)]}{4(\Sigma_0 - \Sigma_\epsilon)}. \square$$

B Proofs for Section 3

Proof of Proposition 1 From equations (33) and (36), we have $\beta(t) = \frac{1}{\sqrt{2(1-\theta)\sigma_s^2(1-t)}}$ and $\frac{1}{\Sigma(t)} = \frac{1}{\sigma_v^2} + \frac{1}{2(1+\theta)\sigma_s^2} + \frac{1}{2(1-\theta)\sigma_s^2(t^{-1}-1)}$, it's clear that both $\Sigma(t)^{-1}$ and $\beta(t)$ are increasing in t and decreasing in σ_s.

Moreover, rewriting the market depth $1/\lambda(t)$ as $2(\Sigma_\epsilon + (\Sigma_0 - \Sigma_\epsilon)t)/(\Sigma_0\sqrt{\Sigma_\epsilon})$ means $1/\lambda(t)$ increases (decreases) with t when $\Sigma_\epsilon < \Sigma_0$ ($\Sigma_\epsilon \geq \Sigma_0$). $\square$

Proof of Proposition 2 When $\rho_0 = 0$, we have

$$\Sigma(t) = \Sigma(0)(1-t)$$

and hence

$$\beta(t) = \frac{1}{\sqrt{\Sigma(0)}(1-t)}, \quad \lambda(t) = \frac{1}{2}, \quad \bar{\lambda}(t) = 1.$$

and the profits of informed investors are

$$\int_0^1 \lambda(t)\, dt = \int_0^1 \frac{1}{2}\, dt = \frac{1}{2}$$

Therefore, market efficiency, market liquidity, and expected profits of informed investors are the same as if there exists a monopolistic informed investor who possesses all private signals in the market. And from equation (29), we have $\Omega(t) = \frac{\Sigma(0)}{2}(1-t)$ and hence $\delta(t) \equiv 1/2$ for all $t \in [0,1]$, which means the conditional correlation between private signals

$\rho(t)$ remains 0 throughout the trading period. $\square$

Proof of Proposition 3 When $\rho_0 \neq 0$, we first have

$$1 \leq 2\delta(t) \leq 1 + \rho_0 \frac{\Omega(t)}{\Omega(0)} \to 1$$

and hence $\rho(t) \to 0$ (as $\delta(t) = (1 + \rho(t))/2$) because $\Omega(t) \leq \Sigma(t) \to 0$ as time t goes to 1. Further,

$$\lim_{t \to 1} \frac{\Sigma(t)}{\Sigma_\epsilon(1-t)} = \lim_{t \to 1} \frac{\Sigma_0}{\Sigma_0 t + \Sigma_\epsilon(1-t)} = 1,$$
$$\lim_{t \to 1} \frac{1/\lambda(t)}{2/\sqrt{\Sigma_\epsilon}} = \lim_{t \to 1} \frac{\Sigma_0 t + \Sigma_\epsilon(1-t)}{\Sigma_0} = 1. \square$$

Proof of Theorem 3 This is Corollary 1 in BCW (2000) and its proof is in the Appendix of BCW (2000). $\square$

Proof of Proposition 4 For $\ln(x) < x - 1$ when $x > 1$ and $\ln(x)/(x-1) \to 0$ as $x \to \infty$, we have $\frac{\Sigma(t)}{\hat{\Sigma}(t)} > 0$ for all $t \in [0, 1]$ and

$$\lim_{t \to 1} \frac{\Sigma(t)}{\hat{\Sigma}(t)} = \lim_{t \to 1} \frac{\ln(1/(1-t))}{t/(1-t)} = 0.$$

Further, the behavior of the ratio of $\beta(t)$ and $\hat{\beta}(t)$ comes directly from the expressions for $\beta(t)$ and $\hat{\beta}(t)$. $\square$

Proof of Proposition 5 The ratio of market liquidity can be decomposed into three components:

$$\frac{1/\lambda(t)}{1/\hat{\lambda}(t)} = \frac{2}{1} \times \frac{\hat{\beta}(t)}{\beta(t)} \times \frac{\hat{\Sigma}(t)}{\Sigma(t)} = 2 \times \sqrt{1-t} \times \frac{\Sigma_\epsilon + \Sigma_0 t/(1-t)}{\Sigma_\epsilon - \Sigma_0 \ln(1-t)}$$

As t approaches 1, $\hat{\beta}(t)/\beta(t)$ goes to zero at the order of $\sqrt{1-t}$ but $\hat{\Sigma}(t)/\Sigma(t)$ goes to infinity at the order of $1/[(1-t)\ln(1-t)]$. Thus, we must have

$$\lim_{t \to 1} \frac{1/\lambda(t)}{1/\hat{\lambda}(t)} = \infty.$$

When $\Sigma_\epsilon \leq \Sigma_0$, we have

$$\begin{aligned}\frac{\hat{\Sigma}(t)}{\Sigma(t)} &= \frac{\Sigma_\epsilon + \Sigma_0 t/(1-t)}{\Sigma_\epsilon - \Sigma_0 \ln(1-t)} \\ &\geq \frac{1}{(1-t)(1-\ln(1-t))} \\ &\geq \frac{\sqrt{e}}{2\sqrt{1-t}},\end{aligned}$$

where the last inequality holds because $\sqrt{1-t}[1-\ln(1-t)]$ is maximized at $t = 1-1/e$. It follows that

$$\begin{aligned}\frac{1/\lambda}{1/\hat{\lambda}} &\geq 2 \times \sqrt{1-t} \times \frac{\sqrt{e}}{2\sqrt{1-t}} = \sqrt{e} > 1, \quad \text{and} \\ \pi_{\mathrm{D}} &= \int_0^1 \lambda(t)dt < \int_0^1 \hat{\lambda}(t)dt = \hat{\pi}_{\mathrm{D}}. \ \square\end{aligned}$$

Proof of Proposition 6

$$\frac{1/\lambda(t)}{1/\hat{\lambda}(t)} = \frac{2}{1} \times \frac{\hat{\beta}(t)}{\beta(t)} \times \frac{\hat{\Sigma}(t)}{\Sigma(t)} = 2 \times \sqrt{1-t} \times \frac{\Sigma_\epsilon + \Sigma_0 t/(1-t)}{\Sigma_\epsilon - \Sigma_0 \ln(1-t)}$$

When $t > 3/4$, $2\sqrt{1-t} < 1$. Moreover, as σ_s increases, $\hat{\Sigma}(t)/\Sigma(t)$ goes to 1 since informed investors have very imprecise signals and thus are reluctant to trade, which causes very little information to be revealed to the market. As a result, market is less liquid in the presence of public disclosure for large σ_s, which means there exists a σ_s^*, such that $1/\lambda(t) < 1/\hat{\lambda}(t)$ for $\sigma_s > \sigma_s^*$ and $t > 3/4$. $\square$

C Proofs for Section 4

Proof of Proposition 7 Redefine $\Sigma_\epsilon = \Sigma_\epsilon/\Sigma_0$, we have

$$\begin{aligned}\frac{\hat{\pi}_{\mathrm{D}}}{\pi_{\mathrm{D}}} &= \int_0^1 \frac{2(\Sigma_\epsilon - 1)/\ln\Sigma_\epsilon}{[\Sigma_\epsilon - \ln(1-t)]\sqrt{1-t}}dt \\ &\leq \int_0^1 \frac{2(\Sigma_\epsilon - 1)/\ln\Sigma_\epsilon}{\Sigma_\epsilon\sqrt{1-t}}dt\end{aligned}$$

$$= \frac{4(\Sigma_\epsilon - 1)}{\Sigma_\epsilon \ln \Sigma_\epsilon}.$$

Remember $\frac{\Sigma_\epsilon}{\Sigma_0} = \frac{(1-\theta)(\sigma_v^2+2(1+\theta)\sigma_s^2)}{(1+\theta)\sigma_v^2}$, so we have $\lim_{\sigma_s\to\infty} \hat{\pi}_{\mathrm{D}}/\pi_{\mathrm{D}} = 0$, which by the definition of limit means there exists a large enough σ_s^{**} such that for $\sigma_s > \sigma_s^{**}$, $\pi_{\mathrm{D}}/\hat{\pi}_{\mathrm{D}} > 1$. □

Proof of Proposition 8

$$\begin{aligned}
\frac{\pi_{\mathrm{D}}}{\pi_{\mathrm{M}}} &= \frac{\Sigma_0\sqrt{\Sigma_\epsilon}(\ln \Sigma_0 - \ln \Sigma_\epsilon)}{4(\Sigma_\epsilon - 1)} \Big/ \frac{\Sigma_0}{2\sqrt{\Sigma_0 + \Sigma_\epsilon}} \\
&= \frac{\sqrt{\Sigma_\epsilon(1+\Sigma_\epsilon)}\ln \Sigma_\epsilon}{2(\Sigma_\epsilon - 1)} (\text{Redefine } \Sigma_\epsilon = \Sigma_\epsilon/\Sigma_0)
\end{aligned}$$

Taking derivative of $\pi_{\mathrm{D}}/\pi_{\mathrm{M}}$ with respect to Σ_ϵ gives

$$\frac{\partial}{\partial \Sigma_\epsilon}\frac{\pi_{\mathrm{D}}}{\pi_{\mathrm{M}}} = \frac{2(\Sigma_\epsilon^2 - 1) - 3(\Sigma_\epsilon + 1)\ln \Sigma_\epsilon}{4(\Sigma_\epsilon - 1)^2\sqrt{\Sigma_\epsilon(1+\Sigma_\epsilon)}}$$

Considering the function $2(\Sigma_\epsilon^2 - 1) - 3(\Sigma_\epsilon + 1)\ln \Sigma_\epsilon$, its first derivative with respect to Σ_ϵ is

$$\begin{aligned}
&\frac{\partial}{\partial \Sigma_\epsilon}\Big[2(\Sigma_\epsilon^2 - 1) - 3(\Sigma_\epsilon + 1)\ln \Sigma_\epsilon\Big] \\
&= 4\Sigma_\epsilon - (3 + 1/\Sigma_\epsilon) - 3\ln \Sigma_\epsilon \\
&\geq 4\Sigma_\epsilon - (3 + 1/\Sigma_\epsilon) - 3(\Sigma_\epsilon - 1) \\
&= \Sigma_\epsilon - 1/\Sigma_\epsilon \geq 0, \Sigma_\epsilon \geq 1.
\end{aligned}$$

and its value is 0 at $\Sigma_\epsilon = 1$, which means $\partial(\pi_{\mathrm{D}}/\pi_{\mathrm{M}})/\partial\Sigma_\epsilon \geq 0$ for all $\Sigma_\epsilon \geq 1$. And also we have the ratio of $\pi_{\mathrm{D}}/\pi_{\mathrm{M}}$ grows to ∞ as Σ_ϵ goes to ∞,

$$\lim_{\Sigma_\epsilon\to\infty}\frac{\pi_{\mathrm{D}}}{\pi_{\mathrm{M}}} = \lim_{\Sigma_\epsilon\to\infty}\frac{\ln \Sigma_\epsilon}{2} = \infty.$$

As $\frac{\Sigma_\epsilon}{\Sigma_0} = \frac{(1-\theta)(\sigma_v^2+2(1+\theta)\sigma_s^2)}{(1+\theta)\sigma_v^2}$, there is a large enough $\bar{\sigma}_s$ such that for $\sigma_s > \bar{\sigma}_s$, we have $\pi_{\mathrm{D}} > \hat{\pi}_{\mathrm{M}} = 2\pi_{\mathrm{M}} > \pi_{\mathrm{M}}$.

Similarly, after redefining $\Sigma_\epsilon = \Sigma_\epsilon/\Sigma_0$, from equation (51) in BCW (2000) we have

$$\begin{aligned}
\frac{\hat{\pi}_{\mathrm{D}}}{\hat{\pi}_{\mathrm{M}}} &= \int_0^1 \frac{\sqrt{\Sigma_\epsilon(1+\Sigma_\epsilon)}/2}{[\Sigma_\epsilon - \ln(1-t)]\sqrt{1-t}}\, dt \\
&\leq \int_0^1 \frac{\sqrt{\Sigma_\epsilon(1+\Sigma_\epsilon)}/2}{[\Sigma_\epsilon + 1 - (1-t)]\sqrt{1-t}}\, dt \quad (\ln(1-t) \leq -t \text{ for } t \in [0,1]) \\
&= \int_0^1 \frac{\sqrt{\Sigma_\epsilon(1+\Sigma_\epsilon)}}{[\Sigma_\epsilon + 1 - s^2]}\, ds \quad (\text{Change of Variable: } s = \sqrt{1-t}) \\
&= \sqrt{\Sigma_\epsilon} \ln \frac{1+\sqrt{1+\Sigma_\epsilon}}{\sqrt{\Sigma_\epsilon}} \\
&< 1. \ \square
\end{aligned}$$

Proof of Corollary 3 In presence (absence) of trade disclosure, the expected profits of an informed investor is π_{M} ($\hat{\pi}_{\mathrm{M}}$) when two investors acquire information on two different stocks and π_{D} ($\hat{\pi}_{\mathrm{D}}$) when two investors acquire information on the same stock. Without trade disclosure, investors will always acquire information on different stocks because $\hat{\pi}_{\mathrm{M}} > \hat{\pi}_{\mathrm{D}}$ always holds. However, in the presence of trade disclosure and when $\sigma_s > \bar{\sigma}_s$, $\pi_{\mathrm{M}} < \pi_{\mathrm{D}}$ holds and hence investors will acquire information on the same stock. $\square$

D Proofs for Section 5

Proof of Theorem 4 We focus on proving the necessity of the claimed equations. The sufficiency of these equations can be established by reversing the necessity arguments (see the end of this proof for more details). So in the rest of this proof except in the last paragraph, we assume that a symmetric linear equilibrium exists, and we prove the claimed equations.

We first prove equations (46) to (49) simply by assuming that each informed investor follows Strategy $\star$. These equations will be used in the inductive proofs for other equations.

First, we can easily check the correctness of equations (46) and (47) by the fact that the expectation of a normal variable is the precision-weighted average of all received signals. Moreover, the updating rule of normally distributed variables states that posterior precision equals prior precision plus the precision of the noise of the signals. Hence, we immediately establish the correctness of equations (48) and (49).

Before proving the rest of the desired equations, we first establish the following useful lemma.

LEMMA 2 *Assume (1) each informed investor believes that all other informed investors follow Strategy $\star$, and (2) the market maker believes that all informed investors follow Strategy $\star$. Then,*

$$\sum_{1\le i\le N}(V_m^i - V_m) = N\delta_m(v - V_m).$$

Proof First, it is easy to check the correctness of the following mathematical identity by properties of normal variables

$$\Omega_0 = \frac{N-1}{N}(1-\rho_0)\Sigma_0. \tag{D1}$$

Using this relation and equations (48) and (49), we can easily check

$$\frac{\Omega_m}{\Omega_0}(N-1)\rho_0 + 1 = N\delta_m. \tag{D2}$$

In what follows, define

$$U_m^i \equiv E[v - s^i - \frac{1}{N}q|F_m^i]$$

where the expectation is computed after trade disclosures in period m. Equivalently, we could have defined $U_m^i \equiv V_m^i - s^i - \frac{1}{N}q$.

Since the expected value of a normal variable is equal to the precision-weighted average of all received signals, we have

$$\begin{aligned} U_m^j &= \frac{\Omega_m}{\Omega_0}U_0^j + \Omega_m \sum_{1\le k\le m}\left[\left(\frac{1}{\Omega_k} - \frac{1}{\Omega_{k-1}}\right)\sum_{i\ne j}\left(s^i + \frac{z_k^i}{\beta_k\Delta t}\right)\right] \\ &= \frac{\Omega_m}{\Omega_0}U_0^j + \Omega_m \sum_{1\le k\le m}\left[\frac{\beta_k^2\Delta t}{(N-1)\sigma_m^2}\sum_{i\ne j}\left(s^i + \frac{z_k^i}{\beta_k\Delta t}\right)\right], \end{aligned} \tag{D3}$$

where the second equation follows from equation (48). (It is easy to verify that equation (48) holds when each informed investor merely *believes* all other informed investors follow Strategy $\star$.) Similarly,

$$V_m = \frac{\Sigma_m}{\Sigma_0}V_0 + \Sigma_m \sum_{1\le k\le m}\left[\frac{\beta_k^2\Delta t}{N\sigma_m^2}\sum_{1\le i\le N}\left(s^i + \frac{z_k^i}{\beta_k\Delta t}\right)\right]. \tag{D4}$$

Summing up equation (D3) over $j = 1, 2, \ldots, N$, we have

$$\begin{aligned}
\sum_{1\le j\le N} U_m^j &= \frac{\Omega_m}{\Omega_0}\Big(\sum_{1\le j\le N} V_0^j - v\Big) + \Omega_m \sum_{1\le k\le m}\left[\frac{\beta_k^2\Delta t}{\sigma_m^2}\sum_{1\le i\le N}\left(s^i + \frac{z_k^i}{\beta_k\Delta t}\right)\right] \\
&= \frac{\Omega_m}{\Omega_0}(N-1)\rho_0 v + N\frac{\Omega_m}{\Sigma_m}V_m \quad \text{(by equation (D4))} \\
&= (N\delta_m - 1)v + N\frac{\Omega_m}{\Sigma_m}V_m \quad \text{(by equation (D2))}.
\end{aligned}$$

The last equation is only a slight variation of the equality claimed in the lemma. □

We have thus completed the proof of Lemma 2. Using the results established in proving the lemma, we next prove that Strategy 41 satisfies equation ($\star$). In equation (41), x_m^i consists of a random component (z_m^i), a component based on public information ($\frac{\beta_m\Delta t}{N\delta_{m-1}}V_{m-1}$), and a private-information-related component ($\frac{\beta_m\Delta t}{N\delta_{m-1}}V_{m-1}^i$). By equation (D3), the only private component in $\frac{\beta_m\Delta t}{N\delta_{m-1}}V_{m-1}^i$ is equal to

$$\frac{\beta_m\Delta t}{N\delta_{m-1}}\left(s^i + \frac{\Omega_{m-1}}{\Omega_0}(N-1)\rho_0 s^i\right) = \beta_m\Delta t s^i \qquad \text{(by equation (D2))}.$$

This proves that Strategy 41 satisfies equation ($\star$). Moreover, our arguments also imply that to support a symmetric linear equilibrium, x_m^i must have the following form:

$$x_m^i - z_m^i = \frac{\beta_m\Delta t}{N\delta_{m-1}}V_{m-1}^i + \text{ a public-information-based component.} \tag{D5}$$

Using Lemma 2 and equation (41), we have

$$\sum_{1\le i\le N} x_m^i = \beta_m\Delta t\left(v - V_{m-1} + \sum_{1\le i\le N}\frac{z_m^i}{\beta_m\Delta t}\right). \tag{D6}$$

Therefore, using equation (47) we immediately obtain equation (43) and equation (44) (the derivation of equation (44) also needs equation (49)). Note that in a symmetric linear equilibrium, the value-updating rules must be of the form specified in equation (43). Our arguments in this paragraph together with equation (D5) also show that to support a symmetric linear equilibrium, equation (41) must hold.

Using equation (D6) and the rules of conditional expectation of normally distributed variables, we immediately obtain equation (42) with

$$\lambda_m = \frac{\mathrm{cov}_{m-1}\left[v, \sum_{1\leq j\leq N} x_m^j + z_m^0\right]}{\mathrm{var}_{m-1}\left[z_m^0 + \sum_{1\leq j\leq N} x_m^j\right]}$$
$$= \frac{\beta_m \Sigma_{m-1}}{\beta_m^2 \Delta t \Sigma_{m-1} + 1 + N\sigma_m^2}.$$

The last equation is exactly equation (45).

We next proceed to prove equations (50) to (59) by backward induction on m, starting with the last period $m = M$. As there are no more trading opportunities after the last period, the maximization problem for each informed investor i is the same as the case without disclosure which has been derived in Foster and Viswanathan (1996) and Cao (1995). In particular, we know that the expected profit function of informed investor i has the form described in equation (50) with the boundary conditions specified in equations (55) to (59).

Thus, we have completed the base step. Next, we assume equations (50) to (54) are correct for period $m+1$ and prove them for period m. By the induction hypothesis, immediately after the mth period disclosure, the expected profits for future trades (i.e., trades from period $m+1$ onwards) can be written as,

$$E_m^i\left[\pi_{m+1}^i\right] \equiv E\left[\pi_{m+1}^i | F_m^i\right] = \alpha_m (V_m^i - V_m)^2 + \zeta_m.$$

Hence, the maximization problem of informed investor i immediately after the $(m-1)$th period trade disclosure is:

$$\max_{x_m^i} E_{m-1}^i\left[x_m^i\left(v - V_{m-1} - \lambda_m\left(z_m^0 + \sum_{1\leq j\leq N} x_m^j\right)\right) + \alpha_m(V_m^i - V_m)^2\right] + \zeta_m \qquad \text{(D7)}$$

where the two terms inside the squared brackets represent the profit of the mth trade and the total profit of all future trades.

For informed investor i to follow a random strategy, he must be indifferent between different values of x_m^i. Thus, the coefficients of $(x_m^i)^2$ and x_m^i in Expression (D7) must be zero. These two restrictions respectively imply

$$\lambda_m = \alpha_m \bar{\lambda}_m^2, \quad \text{and} \tag{D8}$$

$$E_{m-1}^i\left[v - V_{m-1} - \lambda_m \sum_{j\neq i} x_m^j\right] = 2\alpha_m \bar{\lambda}_m E_{m-1}^i\left[V_m^i - V_{m-1} - \bar{\lambda}_m \sum_{j\neq i} x_m^j\right]. \tag{D9}$$

Note that equation (D8) is the same as equation (51). In what follows, we show that equations (D8) and (D9) together imply equation (52). On the other hand, by Lemma 2,

$$\sum_{j\neq i} x_m^j = \beta_m \Delta t \left(v - V_{m-1} - \frac{1}{N\delta_{m-1}}(V_{m-1}^i - V_{m-1})\right) + \sum_{j\neq i} z_m^j. \tag{D10}$$

Hence,

$$E_{m-1}^i\left[\sum_{j\neq i} x_m^j\right] = \beta_m \Delta t \left(1 - \frac{1}{N\delta_{m-1}}\right)\left(V_{m-1}^i - V_{m-1}\right)$$

Applying this relation to equation (D9), we obtain

$$\frac{1 - \lambda_m \beta_m \Delta t + \lambda_m \beta_m \Delta t/(N\delta_{m-1})}{1 - \bar{\lambda}_m \beta_m \Delta t + \bar{\lambda}_m \beta_m \Delta t/(N\delta_{m-1})} = 2\alpha_m \bar{\lambda}_m.$$

Now we multiply both sides of the preceding equation with the denominator of the left-hand side of the equation, and then we use equation (D8) to substitute all the $\alpha_m \bar{\lambda}^2$ terms by λ_m. This leads to

$$2\alpha_m \bar{\lambda}_m = 1 + \lambda_m \left(\beta_m \Delta t - \frac{\beta_m \Delta t}{N\delta_{m-1}}\right).$$

Next, multiplying both sides of the above equation with $\bar{\lambda}_m$ and using equation (D8) to substitute $\alpha_m \bar{\lambda}^2$ by λ_m, we immediately obtain equation (52).

Since we have established that informed investor i is indifferent to x_m^i, Expression (D7) can be simplified by setting $x_m^i = 0$. Thus,

$$\begin{aligned} E_{m-1}^i[\pi_m^i] &= \alpha_m E_{m-1}^i\left[\left(V_m^i - V_m\right)^2\right] + \zeta_m \\ &= \alpha_m \left(E_{m-1}^i\left[V_m^i - V_m\right]\right)^2 + \alpha_m \mathrm{var}_{m-1}^i[V_m^i - V_m] + \zeta_m \end{aligned} \tag{D11}$$

On the other hand, since we have assumed $x_m^i = 0$ in the profit calculation, using the updating rule for normal variables we have

$$\begin{aligned} V_m^i &= \frac{\Omega_m}{\Omega_{m-1}} V_{m-1}^i + \frac{\Omega_{m-1} - \Omega_m}{\Omega_{m-1}} \left(v + \sum_{j \neq i} \frac{z_m^j}{\beta_m \Delta t} \right) \\ &= \frac{\Omega_m}{\Omega_{m-1}} V_{m-1}^i + \frac{\Omega_m \beta_m^2 \Delta t}{(N-1)\sigma_m^2} \left(v + \sum_{j \neq i} \frac{z_m^j}{\beta_m \Delta t} \right), \end{aligned} \tag{D12}$$

where the second equation follows from equation (48). Moreover, using the pricing rules by market maker and applying equation (D10), we have

$$\begin{aligned} V_m &= V_{m-1} + \bar{\lambda}_m \beta_m \Delta t \left(v - V_{m-1} - \frac{V_{m-1}^i - V_{m-1}}{N \delta_{m-1}} \right) + \bar{\lambda}_m \sum_{j \neq i} z_m^j \\ &= V_{m-1} + \frac{\beta_m^2 \Sigma_m \Delta t}{N \sigma_m^2} \left(v - V_{m-1} - \frac{V_{m-1}^i - V_{m-1}}{N \delta_{m-1}} + \sum_{j \neq i} \frac{z_m^j}{\beta_m \Delta t} \right), \end{aligned} \tag{D13}$$

where the second equation follows from equation (44).

Now, using equations (D12) and (D13) and the fact that v is independent of $\sum_{j \neq i} z_m^j$, we have

$$\mathrm{var}_{m-1}^i \left[V_m^i - V_m \right] = \left(\frac{\Omega_m \beta_m^2 \Delta t}{(N-1)\sigma_m^2} - \frac{\beta_m^2 \Sigma_m \Delta t}{N \sigma_m^2} \right)^2 \left(\Omega_{m-1} + \frac{(N-1)\sigma_m^2}{\beta_m^2 \Delta t} \right) \tag{D14}$$

Moreover,

$$\begin{aligned} E_{m-1}^i [V_m^i - V_m] &= V_{m-1}^i - E_{m-1}^i [V_m] \\ &= \left(1 - \frac{\beta_m^2 \Sigma_m \Delta t}{N \sigma_m^2} \left(1 - \frac{1}{N \delta_{m-1}} \right) \right) (V_{m-1}^i - V_{m-1}), \end{aligned} \tag{D15}$$

here the last equation follows from equation (D13). Substituting equations (D14) and (D15) into equation (D11), we immediately see that equation (50) is correct for m with α and ζ satisfying equations (53) and (54). This completes our inductive step.

So far, we have proved all the desired equations as necessary conditions to support a symmetric linear equilibrium. In proving these equations, we have used (1) the rationality of the market maker's pricing rules and value-updating rules, and (2) the optimality of all informed investors' trading strategies. Moreover, by reversing these arguments, we can easily check that when these equations indeed hold, (1) the pricing rules and value-updating rules are indeed rational for the market maker, and (2) the trading strategies of all informed investors are indeed optimal. Therefore, all these equations collectively form a set of sufficient conditions to support a symmetric linear equilibrium. □

Numerical method in solving the system of equations in Theorem 4

The whole recursive system of α_m, β_m, λ_m, $\bar{\lambda}_m$, Σ_m, Ω_m, and ζ_m can be numerically solved by first conjecturing a value of Ω_{M-1} and then solving recursively for $\Omega_{M-2}, \ldots, \Omega_0$. Given the conjectured Ω_{M-1}, we can compute δ_{M-1}, since the definition of δ_M and equations (48) and (49) imply

$$N\delta_{M-1} = 1 + \frac{\Omega_{M-1}}{\Omega_0}(N\delta_0 - 1).$$

From Ω_{M-1} and δ_{M-1}, we can now derive Σ_{M-1}. From the boundary condition in equation (57), we can determine α_{M-1}. Now again we conjecture a value for Ω_{M-2}, which allows us to derive δ_{M-2} and Σ_{M-2} as before. From equations (44) and (49),

$$\Sigma_{M-1}^{-1} = \Sigma_{M-2}^{-1} + \bar{\lambda}_{M-1}\beta_{M-1}\Delta t/\Sigma_{M-1}.$$

Consequently, we obtain $\beta_{M-1}\bar{\lambda}_{M-1}$. Comparing equation (45) and equation (51), we arrive at

$$\beta_{M-1}\Sigma_{M-2}/(\beta_{M-1}^2\Delta t\Sigma_{M-2} + 1 + N\sigma_{M-1}^2) = \bar{\lambda}_{M-1}^2\alpha_{M-1}.$$

In the preceding equation, we can use the derived expression for $\beta_{M-1}\bar{\lambda}_{M-1}$ to substitute $\bar{\lambda}_{M-1}$ for β_{M-1}, and we can use equation (44) to substitute $\bar{\lambda}_{M-1}$ for σ_{M-1}^2. Doing so results in an equation with $\bar{\lambda}_{M-1}$ being the only unknown. Solving the resulting equation gives a formula for $\bar{\lambda}_{M-1}$. Next we can derive β_{M-1} from $(\beta_{M-1}\bar{\lambda}_{M-1})/\bar{\lambda}_{M-1}$, λ_{M-1} from equation (51), and σ_{M-1}^2 from equation (44). Given the expressions for λ_{M-1}, $\bar{\lambda}_{M-1}$, β_{M-1}, and σ_{M-1}^2, we can now check whether equation (52) holds or not. If it doesn't, we modify our initial value of Ω_{M-2} until it holds. We repeat the procedure to derive $\Omega_{M-3}, \ldots, \Omega_0$. If the derived Ω_0 is different from the initial given value, we adjust Ω_{M-1} and repeat the whole procedure until the derived Ω_0 equals to the initial given value. □

Proof of Theorem 5 Before proving Theorem 5, we prove the following general version of Corollary 2.

PROPOSITION 11 *Ignoring the higher order terms of Δt, we have the following results* *for* $m = 1, \ldots, M-1$

$$\bar{\lambda}_m = \beta_m \Sigma_m \tag{D16}$$

$$\lambda_m = \beta_m \Sigma_m / 2 \tag{D17}$$

$$\sigma_m^2 = 1/N \tag{D18}$$

$$\frac{\Delta \Omega_m^{-1}}{\Delta t} = \frac{\Omega_m^{-1} - \Omega_{m-1}^{-1}}{\Delta t} = \frac{N\beta_m^2}{N-1} \tag{D19}$$

$$\frac{\Delta \Sigma_m^{-1}}{\Delta t} = \frac{\Sigma_m^{-1} - \Sigma_{m-1}^{-1}}{\Delta t} = \beta_m^2 \tag{D20}$$

$$\bar{\lambda}_m = \frac{1}{2\alpha_m} \tag{D21}$$

$$\frac{\Delta \alpha_m}{\Delta t} = \frac{\alpha_m - \alpha_{m-1}}{\Delta t} = 2\alpha_m \beta_m^2 \Sigma_m \left(1 - \frac{1}{N\delta_{m-1}}\right) \tag{D22}$$

$$\frac{\Delta \zeta_m}{\Delta t} = \frac{\zeta_m - \zeta_{m-1}}{\Delta t} = -\alpha_m \beta_m^2 \frac{(N\Omega_m - (N-1)\Sigma_m)^2}{N(N-1)}. \tag{D23}$$

Proof From equation (52), as $\Delta t \to 0$ we have $\lambda_m = \bar{\lambda}_m/2$ and equation (D21) comes directly from equation (51).

Combining equations (44), (45) and (49) gives

$$\begin{aligned}\frac{\bar{\lambda}_m}{\lambda_m} &= \frac{\beta_m(N\sigma_m^2\Sigma_{m-1})/(N\sigma_m^2 + \beta_m^2\Sigma_{m-1}\Delta t)}{N\sigma_m^2} \frac{1 + N\sigma_m^2 + \beta_m^2\Sigma_{m-1}\Delta t}{\beta_m \Sigma_{m-1}} \\ &= \frac{1 + N\sigma_m^2 + \beta_m^2\Sigma_{m-1}\Delta t}{N\sigma_m^2 + \beta_m^2\Sigma_{m-1}\Delta t} = 2\end{aligned}$$

which means $N\sigma_m^2 = 1$ as $\Delta t \to 0$. So we have the total trading volume from informed investors is $N\sigma_m^2 = 1$, which equals the trading volume from noise traders. Therefore, informed investors contribute half of the trading volume in the market with disclosure.

The proof of equations (D19) and (D20) is trivial. Equations (D22) and (D23) comes directly from equations (53) and (54) and $\sigma_m^2 = 1/N$. □

Combining equation (51), equation (56), and equation (57) gives us

$$\frac{\lambda_{M-1}}{\bar{\lambda}_{M-1}^2}(1 + N\delta_{M-1})\sqrt{N\delta_{M-1}\Sigma_{N-1}} = \sqrt{\Delta t}.$$

From equation (D2), we know $N\delta_{M-1} > \min\{1, 1 + (N-1)\rho_0\} > 0$. As $\Delta t \to 0$,

$\lambda_{M-1}/\bar{\lambda}_{M-1} = 1/2$, so must have

$$\Sigma_{M-1} \to 0, \text{ or} \tag{D24a}$$
$$\bar{\lambda}_{M-1} \to +\infty. \tag{D24b}$$

Let $\Gamma(t) = \Sigma^{-1}(t)$, then we have

$$\beta(t) = \sqrt{\frac{d\Gamma(t)}{dt}} = \sqrt{\Gamma'(t)}, \qquad \text{(by equation (D20))}$$
$$\bar{\lambda}(t) = \beta(t)\Sigma(t) = \sqrt{\Gamma'(t)}/\Gamma(t). \qquad \text{(by equation (44))}$$

Define $A \equiv \frac{N\rho_0}{(1-\rho_0)\Sigma_0} = \frac{N\delta_0 - 1}{\Omega_0}$, then we have

$$\begin{aligned}
\Omega(t) &= \frac{N-1}{N/\Sigma(t) + (N\delta_0 - 1)/\Omega(0)} \qquad \text{(by equations (D19) and (D20))} \\
&= \frac{N-1}{A + N\Gamma(t)}, \\
1 - \frac{1}{N\delta(t)} &= 1 - \frac{1}{1 + \frac{N\delta_0 - 1}{\Omega_0}\Omega(t)} \qquad \text{(by equation (D2))} \\
&= 1 - \frac{1}{1 + A\frac{N-1}{A+N\Gamma(t)}} \\
&= \frac{(N-1)A}{N(A+\Gamma(t))}.
\end{aligned}$$

Plugging the above equation into equation (D22) gets

$$\alpha'(t) = \beta(t)\left(1 - \frac{1}{N\delta(t)}\right) = \frac{(N-1)A\sqrt{\Gamma'(t)}}{N(A+\Gamma(t))}$$

And at the same time, we have

$$\begin{aligned}
\alpha'(t) &= \left(\frac{1}{2\beta(t)\Sigma(t)}\right)' = \left(\frac{\Gamma(t)}{2\sqrt{\Gamma'(t)}}\right)' \\
&= \frac{1}{2}\left((\Gamma'(t))^{\frac{1}{2}} - \frac{1}{2}\Gamma(t)(\Gamma'(t))^{-\frac{3}{2}}\Gamma''(t)\right)
\end{aligned}$$

which means $\Gamma(t)$ satisfies the following ordinary differential equation

$$\frac{\Gamma''(t)}{\Gamma'(t)} + \left(2 - \frac{4}{N}\right)\frac{\Gamma'(t)}{\Gamma(t)} + \frac{4(N-1)\Gamma'(t)}{N(-A-\Gamma(t))} = 0 \tag{D25}$$

with initial condition $\Gamma(0) = \Sigma(0)^{-1}$ and terminal condition (D24).

In the case $N > 1$ and $\rho_0 = 1$, $A = \infty$, and hence the above equation implies[10]

$$0 = \frac{d}{dt}\left[\ln\left(\Gamma'(t)\Gamma(t)^{2-\frac{4}{N}}\right)\right].$$

Thus,

$$\Gamma'(t)\Gamma(t)^{2-\frac{4}{N}} = C_0 \text{ for some constant } C_0 > 0,$$

which in turns implies

$$\Sigma(t) = \frac{1}{\Gamma(t)} = (C_1 t + C_2)^{\frac{-1}{3-\frac{4}{N}}} \text{ for some constants } C_1 \text{ and } C_2. \tag{D26}$$

But when $N > 1$, there are no constants $C_1 = C_0(3 - 4/N) > 0$ and C_2 that can make the above $\Sigma(t)$ satisfy either $\Sigma(1) = 0$ or $\lim_{t\to 1}\bar{\lambda}(t) = \sqrt{-\Sigma'(1)} = +\infty$, as required by equation (D24). This completes the proof that a linear equilibrium does not exist for $N > 1$ and $\rho_0 = 1$.

In the rest of the proof, we assume either $\rho_0 \neq 1$ or $N = 1$. Under these assumptions, we prove that equation (D25) has a unique solution of $\Sigma(t)$ as described in the theorem. Now, the only possible case with $\rho_0 = 1$ happens is when $N = 1$. But when $N = 1$, there is no competing informed investors, and ρ_0 is irrelevant. Without loss of generality, we make the additional assumption $\rho_0 \neq 1$. This will ensure a finite A in the rest of the proof.

By equation (D25),

$$0 = \frac{d}{dt}\left[\ln\left(\Gamma'(t)\Gamma(t)^{2-\frac{4}{N}}(\Gamma(t)+A)^{-\frac{4(N-1)}{N}}\right)\right].$$

Hence,

$$\Gamma'(t)\Gamma(t)^{2-\frac{4}{N}}\left(\Gamma(t)+A\right)^{\frac{-4(N-1)}{N}} = C_3 \text{ for some constant } C_3,$$

In the case of $\rho_0 = 0$, we have $A = 0$. Hence, the above equation is equivalent to

[10] To be completely formal and to avoid dividing by 0, we should have directly derived the desired equation below. But this is a straightforward exercise by using the argument for obtaining equation (D25).

$\Gamma^{-2}(t)\Gamma'(t) = C_3$, which implies that $\Sigma(t) = 1/\Gamma(t)$ is linear in t. Hence, the desired formula for Σ follows immediately from the boundary condition $\Sigma(1) = 0$.

For the case of $\rho_0 \neq 0$, we can make a change of variable as $\Gamma(t) = A\frac{R(t)}{1-R(t)}$, the above equation becomes

$$R(t)^{2-\frac{4}{N}} R'(t) = C_4.$$

From this and the boundary condition on $R(0)$ and $R(1)$, we obtain

$$\frac{1}{A\Sigma(t)+1} = \left(\left[\left(\frac{1}{A\Sigma(1)+1} \right)^{3-\frac{4}{N}} - B \right] t + B \right)^{\frac{1}{3-\frac{4}{N}}}. \tag{D27}$$

Taking derivatives with respect to t in the above equation, we know that $\Sigma'(1)$ is bounded. Hence, from the proved formula $\bar{\lambda}(t) = \sqrt{-\Sigma'(t)}$, we know that $\lim_{t\to 1} \bar{\lambda}$ is finite. Hence, from equation (D24), we must have $\Sigma(1) = 0$. Plugging $\Sigma(1) = 0$ into equation (D27), we immediately arrive at the claimed formula for $\Sigma(t)$.

We first prove the case of $N \neq 1$ and $\rho_0 \neq 0$. From equation (50), we know that the **ex ante** expected profit of a single informed trader is $\mathrm{E}_0[\pi_1^i] = \mathrm{E}_0[\alpha_0(V_0^i - V_0)^2] + \zeta_0$. The first term is

$$\mathrm{E}_0\left[\frac{((1+(N-1)\rho_0)s^i)^2}{2\bar{\lambda}_0}\right] = \frac{(1+(N-1)\rho_0)\Sigma_0}{2N(a_N A(1-B)^{-1}B^{(1+a_N^{-1})})^{-\frac{1}{2}}} \tag{D28}$$

and the second term is

$$\begin{aligned}
\zeta(0) &= \zeta(1) + \int_0^1 \frac{1}{\bar{\lambda}(u)} \left(\bar{\lambda}(u) - \frac{N\beta(u)\Omega(u)}{N-1} \right)^2 du \\
&= \int_0^1 \bar{\lambda}(u) \left(1 + N\Gamma(u)/A\right)^{-2} du \\
&= -\int_0^1 \frac{A\bar{\lambda}(u)}{N\beta^2(u)} d\left(1 + N\Gamma(u)/A\right)^{-1} \\
&= \frac{A}{N}\left[\frac{\rho_0\bar{\lambda}_0}{\beta_0^2} - \frac{2}{N}\int_0^1 \left(1+N\Gamma(u)/A\right)^{-1} \frac{\beta^2(u)\Sigma(u)}{1-\Omega(u)/\Sigma(u)}\right] du \\
&= \frac{A}{N}\left[\frac{\rho_0\bar{\lambda}_0}{\beta_0^2} - \frac{2}{N}\int_0^1 \frac{\beta^2(u)\Sigma(u)}{1+\Gamma(u)/A} du\right]
\end{aligned}$$

$$
\begin{aligned}
&= \frac{A}{N}\left\{\frac{\rho_0\bar{\lambda}_0}{\beta_0^2} - \frac{4(a_N A(1-B)^{-1}B^{\frac{3}{a_N}})^{\frac{1}{2}}}{NA(B-1)}\right. \\
&\times \left.\left[\frac{2B^{\frac{1}{a_N}+\frac{1}{2}}}{a_N^{-1}-1} + \frac{B^{\frac{2}{a_N}+\frac{1}{2}}}{a_N^{-1}+1} - \frac{B^{-\frac{1}{2}}}{3a_N^{-1}-1} - \frac{8a_N^{-2}B^{\frac{3}{2a_N}}}{(a_N^{-2}-1)(3a_N^{-1}-1)}\right]\right\}
\end{aligned}
\tag{D29}
$$

here, $a_N = 3 - 4/N$ and B is defined in Theorem 5. It's straightforward to check that the sum of equations (D28) and (D29) is exactly equation (60).

In the case of $N = 1$ or $\rho_0 = 0$, we have $\zeta(0) = 0$ and equation (D28) becomes

$$
\frac{\Sigma_0}{2N\sqrt{\Sigma_0}} = \frac{\sqrt{\Sigma_0}}{2N},
$$

which completes the proof. □

Proof of Theorem 6 The proof is in the Appendix of BCW (2000). □

Proof of Proposition 9

(i) From equation (D26), when $N = 1$, we have $\Sigma(t) = C_1 t + C_2$ for some constants C_1, C_2. And only $C_1 = -\Sigma_0, C_2 = \Sigma_0$ satisfies the initial condition and the condition $\lim_{t\to 1}\Sigma(t) = 0$ or $\lim_{t\to 1}\hat{\lambda}(t) = +\infty$, required by equation (D24). Thus, we show

$$
\Sigma(t) = \Sigma(0)(1-t) = \hat{\Sigma}(t).
$$

and it's trivial to show $\beta(t) = \hat{\beta}(t), \lambda(t) = \hat{\lambda}(t)/2$.

(ii) For ease of notation, we define

$$
b_{N\delta} = \frac{2(1-\delta_0)}{N\delta_0}.
$$

Rewriting $\Sigma(t)$ as

$$
\Sigma(t) = \frac{((1-B)t+B)^{-1/a_N}-1}{B^{-1/a_N}-1}\Sigma(0).
$$

and its derivative with respect to t as

$$
\frac{\partial\Sigma(t)}{\partial t} = -\frac{\Sigma(0)(1-B)((1-B)t+B)^{-1-1/a_N}}{a_N(B^{-1/a_N}-1)} \tag{D30}
$$

Differentiating both sides of equation (62) gives

$$\frac{\partial\hat{\Sigma}(t)}{\partial t} = -\kappa\hat{\Sigma}(0)^{-a_N}\hat{\Sigma}(t)^{1+a_N}e^{b_{N\delta}\hat{\Sigma}(0)/\hat{\Sigma}(t)}$$

The information is gradually revealed as time flows and the public knows exactly the liquidation value v at the end of the trading period in both cases with disclosure and without disclosure. Both $\Sigma(t)$ and $\hat{\Sigma}(t)$ goes to 0 as $t \to 1$, so the L'Hospital's Rule is applied to calculate the following limit:

$$\begin{aligned}
\lim_{t\to 1}\frac{\Sigma(t)}{\hat{\Sigma}(t)} &= \lim_{t\to 1}\frac{\partial\Sigma(t)/\partial t}{\partial\hat{\Sigma}(t)/\partial t} \\
&= \lim_{t\to 1}\frac{-\Sigma(0)(1-B)((1-B)t+B)^{-1-1/a_N}/[a_N(B^{-1/a_N}-1)]}{-\kappa\hat{\Sigma}(0)^{-a_N}\hat{\Sigma}(t)^{1+a_N}e^{b_{N\delta}\hat{\Sigma}(0)/\hat{\Sigma}(t)}} \\
&= \frac{\Sigma(0)\hat{\Sigma}(0)^{a_N}(1-B)}{\kappa a_N(B^{-1/a_N}-1)}\lim_{t\to 1}\frac{((1-B)t+B)^{-1-1/a_N}-1}{\hat{\Sigma}(t)^{1+a_N}e^{b_{N\delta}\hat{\Sigma}(0)/\hat{\Sigma}(t)}} \\
&= 0
\end{aligned} \tag{D31}$$

The exponential function $e^{b_{N\delta}\hat{\Sigma}(0)/\hat{\Sigma}(t)}$ grows much faster than the polynomial function $\hat{\Sigma}(t)^{-1-a_N}$ as $\hat{\Sigma}(t)$ goes to 0, so the denominator $\hat{\Sigma}(t)^{1+a_N}e^{b_{N\delta}\hat{\Sigma}(0)/\hat{\Sigma}(t)}$ goes to ∞ and the numerator $((1-B)t+B)^{-1-1/a_N}-1$ goes to 0 as time $t \to 1$, which proves the last equation.

Similarly, we have the following result

$$\begin{aligned}
\lim_{t\to 1}\frac{\beta(t)}{\hat{\beta}(t)} &= \lim_{t\to 1}\frac{\sqrt{-\Sigma'}/\Sigma}{\sqrt{-\hat{\Sigma}'}/\hat{\Sigma}} \\
&= \frac{\lim_{t\to 1}\sqrt{\Sigma'/\hat{\Sigma}'}}{\lim_{t\to 1}\Sigma'/\hat{\Sigma}'} \quad \text{(By L'Hospital's Rule)} \\
&= \lim_{t\to 1}(\Sigma'/\hat{\Sigma}')^{-\frac{1}{2}} \\
&= \infty \quad \text{(By equation (D31))}
\end{aligned}$$

(iii) Similarly, we have

$$\lim_{t\to 1}\frac{1/\lambda}{1/\hat{\lambda}} = \lim_{t\to 1}\frac{2/\sqrt{-\Sigma'}}{1/\sqrt{-\hat{\Sigma}'}}$$

$$
\begin{aligned}
&= \lim_{t\to 1} \frac{2}{\sqrt{\Sigma'/\hat{\Sigma}'}} \\
&= \infty \quad \text{(By equation (D31))}
\end{aligned}
$$

(iv) As time $t \to 1$, more and more information is revealed and the uncertainty about the liquidity value v decreases. This can be seen clearly from equation (D30). Given $a_N \geq 1$ for $N > 1$, $B - 1$ and $B^{-1/a_N} - 1$ take opposite signs, so we have $\partial\Sigma(t)/\partial t < 0$.

For B is an increasing function of σ_ϵ, we here prove the function's monotonicity with respect to B rather than σ_ϵ. Taking derivative of $\Sigma(t)$ with respect to B, we get

$$
\frac{\partial\Sigma(t)}{\partial B} \propto (1 - B^{-1-1/a_N})(1-t) + B^{-1-1/a_N} - (t/B + 1 - t)^{1+1/a_N}
$$

The derivative of $(1 - B^{-1-1/a_N})(1-t) + B^{-1-1/a_N} - (t/B + 1 - t)^{1+1/a_N}$ with respect to B is $(1/a_N + 1)B^{-2-1/a_N}t[(t + B(1-t))^{1/a_N} - 1]$, which is larger than 0 if $B \geq 1$ and smaller than 0 if $B < 1$. So $(1 - B^{-1-1/a_N})(1-t) + B^{-1-1/a_N} - (t + B(1-t))^{1+/a_N}$ reaches its minimum 0 at $B = 1$ and we have $\partial\Sigma(t)/\partial B \geq 0$ for all $B > 0$.

From the definition of $\lambda(t)$, we have

$$
\begin{aligned}
\frac{\partial\lambda(t)}{\partial t} &= \frac{\partial(-\Sigma^{'})/\partial t}{2\lambda(t)} \\
&= \frac{\Sigma(0)(1 + 1/a_N)(1-B)^2((1-B)t + B)^{-2-1/a_N}}{8\lambda(t)a_N(B^{-1/a_N} - 1)} \\
&\propto 1/(1 - B^{1/a_N})
\end{aligned}
$$

So, $\lambda(t)$ is increasing in t when $B < 1$ ($\rho_0 > 0$) and decreasing in t when $B \geq 1$ ($\rho_0 \leq 0$).

(v) When $\rho_0 = 0$, we have

$$
\Sigma(t) = \Sigma(0)(1-t)
$$

and hence

$$
\beta(t) = \frac{1}{\sqrt{\Sigma(0)}(1-t)}, \quad \lambda(t) = \frac{1}{2}, \quad \bar{\lambda}(t) = 1.
$$

and the profits of informed investors are

$$\int_0^1 \lambda(t)\,dt = \int_0^1 \frac{1}{2}\,dt = \frac{1}{2}$$

Therefore, market efficiency, market liquidity, and expected profits of informed investors are the same as if there exists a monopolistic informed investor with all the signals in the market.

We have $\Sigma(0) = N\Omega(0)/(N-1)$ when $\rho_0 = 0$, so the equation $\frac{1}{\Sigma(t)} - \frac{1}{\Sigma(0)} = \frac{N}{N-1}\left(\frac{1}{\Omega(t)} - \frac{1}{\Omega(0)}\right)$ means $\delta(t) \equiv 1/N$ and hence equation (40) (i.e., $\delta(t) = (1+(N-1)\rho(t))/N$) in BCW (2000) means $\rho(t) \equiv 0$, that is the conditional correlation between private signals $\rho(t)$ remains 0 throughout the trading period.

(vi) When $\rho_0 \neq 0$, we first have

$$N\delta(t) = 1 + (N-1)\rho_0\frac{\Omega(t)}{\Omega(0)} \to 1$$

and hence $\rho(t) \to 0$ (from equation (40) in BCW (2000)) because $\Omega(t) \leq \Sigma(t) \to 0$ as time t goes to 1. Further,

$$\begin{aligned}
\lim_{t\to 1}\frac{\Sigma(t)}{1-t} &= \lim_{t\to 1} -\Sigma'(t) \quad \text{(By L'Hospital's Rule)} \\
&= \frac{(1-\rho_0)\Sigma(0)(1-B)}{\rho_0 N a_N}\lim_{t\to 1}((1-B)t+B)^{-1-1/a_N} \\
&= \frac{(1-\rho_0)\Sigma(0)(1-B)}{\rho_0 N a_N} \\
&= S_0
\end{aligned}$$

from here we also have $\lim_{t\to 1}\Sigma^{'}(t) = -S_0$.

It's easy to verify

$$\begin{aligned}
\lim_{t\to 1}\beta(t)(1-t) &= \lim_{t\to 1}\frac{\sqrt{-\Sigma^{'}(t)}}{\Sigma(t)/(1-t)} \\
&= \frac{\lim_{t\to 1}\sqrt{-\Sigma^{'}(t)}}{\lim_{t\to 1}\Sigma(t)/(1-t)} \\
&= \frac{\sqrt{S_0}}{S_0}
\end{aligned}$$

$$
\begin{aligned}
&= 1/\sqrt{S_0}, \quad \text{and} \\
\lim_{t\to 1}\lambda(t) &= \lim_{t\to 1}\frac{\sqrt{-\Sigma'(t)}}{2} \\
&= \frac{\lim_{t\to 1}\sqrt{-\Sigma'(t)}}{2} \\
&= \frac{\sqrt{S_0}}{2}
\end{aligned}
$$

Writing the profit $\pi(0)$ as a function of B, we have

$$
\begin{aligned}
\pi(0) &= \sqrt{\frac{a_N\Sigma(0)}{4(N-2)^2}\frac{(1-B^{(1-1/a_N)/2})^2}{(1-B)(B^{-1/a_N}-1)}}, \quad \text{and} \\
\frac{\partial\pi(0)}{\partial B} &\propto \frac{\partial}{\partial B}\left[\frac{(1-B^{(1-1/a_N)/2})^2}{(1-B)(B^{-1/a_N}-1)}\right] \\
&= -\frac{B^{(1/a_N-1)/2}(1-B^{(1-1/a_N)/2})(1-B^{(1+1/a_N)/2})}{a_N(1-B)^2(1-B^{1/a_N})^2} \\
&\quad \times\left[a_N(1-B^{1/a_N})-(1-B)B^{(1/a_N-1)/2}\right] \\
&\propto a_N(B^{1/a_N}-1)-(B-1)B^{(1/a_N-1)/2}
\end{aligned}
$$

Again, taking derivative of $a_N(B^{1/a_N}-1)-(B-1)B^{(1/a_N-1)/2}$ with respect to B gives $B^{(1/a_N-3)/2}\left[B^{(1/a_N+1)/2}-(1/a_N+1)B/2+(1/a_N-1)/2\right]$. The derivative of the second term $B^{(1/a_N+1)/2}-(1/a_N+1)B/2+(1/a_N-1)/2$ is $(1/a_N+1)(B^{(1/a_N-1)/2}-1)/2$. Given $a_N \geq 1$, $B^{(1/a_N-1)/2}-1$ is negative if $B>1$ and positive if $B\leq 1$, so the derivative of $a_N(B^{1/a_N}-1)-(B-1)B^{(1/a_N-1)/2}$ with respect to B reaches its maximum 0 at $B=1$, which means it decreases in B and equals to 0 at $B=1$. So, $\partial\pi(0)/\partial B$ is positive when $B<1$ ($\rho_0>0$) and negative when $B\geq 1$ ($\rho_0\leq 0$), i.e., $\pi(0)$ reaches its maximum at $B=1$ ($\rho_0=0$). □

Proof of Proposition 10 When one of the N informed investors (without loss of generality, assume she is the N-th trader) leaves the market, the remaining $N-1$ investors in aggregate don't know the true value of the asset v. Instead, the variable that the $N-1$ informed investors and the market maker are interested in is informed investors' expectation of v:

$$
v_{N\to N-1} = E[v|q,s_1,\ldots,s_{N-1}] = \frac{\sigma_v^2((1+(N-2)\theta)q+(1+(N-1)\theta)\sum_{i=1}^{N-1}s^i)}{(1+(N-2)\theta)\sigma_v^2+(1-\theta)(1+(N-1)\theta)\sigma_s^2}.
$$

Correspondingly, the expected profit each of the remaining $N-1$ informed investors obtains

$$\pi_{N\to N-1} = \sqrt{\Sigma_{N\to N-1}(0)\frac{1-\rho_0}{\rho_0}\frac{3(N-1)-4}{1-B_{N\to N-1}}}\ \frac{\left|1-B_{N\to N-1}^{\frac{(N-1)-2}{3(N-1)-4}}\right|}{2(N-1)|(N-1)-2|}$$

here,

$$\begin{aligned}
\Sigma_{N\to N-1}(0) &= \mathrm{var}[v_{N\to N-1}|q] = \frac{(N-1)^2\Sigma_0}{(N-1)^2+\Sigma_\epsilon/\Sigma_0} \\
&\sim O\left(\frac{(N-1)^2\Sigma_0^2}{\Sigma_\epsilon}\right), \quad \sigma_s \to \infty \\
B_{N\to N-1} &= \left(\frac{1-\rho_0}{1-\rho_0+(N-1)\rho_0}\right)^{3-4/(N-1)} = \left(\frac{N\Sigma_\epsilon}{(N-1)^2\Sigma_0+\Sigma_\epsilon}\right)^{3-4/(N-1)} \\
&\sim O\left(N^{3-4/(N-1)}\right), \quad \sigma_s \to \infty.
\end{aligned}$$

Considering the limiting behavior of the ratio of π_N and $\pi_{N\to N-1}$ when $\sigma_s \to \infty$:

$$\begin{aligned}
&\lim_{\sigma_s\to\infty}\frac{\pi_N^2}{\pi_{N\to N-1}^2} \\
&= \lim_{\sigma_s\to\infty}\frac{\lim_{\sigma_s\to\infty}\pi_N^2}{\lim_{\sigma_s\to\infty}\pi_{N\to N-1}^2} \\
&= \lim_{\sigma_s\to\infty}\frac{-\Sigma_0\frac{1-\rho_0}{\rho_0}\frac{(3N-4)\Sigma_0}{\Sigma_\epsilon/(N-1)}/(4N^2(N-2)^2)}{\frac{(N-1)^2\Sigma_0^2}{\Sigma_\epsilon}\frac{1-\rho_0}{\rho_0}\frac{3(N-1)-4}{1-N^{3-4/(N-1)}}\frac{(1-N^{1-2/(N-1)})^2}{4(N-1)^2((N-1)-2)^2}} \\
&= \frac{(3N-4)(N-1)(N-3)^2(N^{3-4/(N-1)}-1)}{(3N-7)N^2(N-2)^2(1-N^{1-2/(N-1)})^2} \\
&> 1, \quad N \geq 4
\end{aligned}$$

In fact, $\lim_{\sigma_s\to\infty}\pi_N^2/\pi_{N\to N-1}^2$ decreases to 1 as N goes to ∞.

For the case of $N=3$, following similar steps, we get

$$\lim_{\sigma_s\to\infty}\frac{\pi_3^2}{\pi_{3\to 2}^2} = \lim_{\sigma_\epsilon^2\to\infty}\frac{\Sigma_0\frac{1-\rho_0}{\rho_0}\frac{5\Sigma_0}{\Sigma_\epsilon/2}/36}{\frac{4\Sigma_0^2}{64\Sigma_\epsilon}\frac{1-\rho_0}{\rho_0}(\ln 3)^2} = \frac{40}{9(\ln 3)^2} = 3.68 > 1$$

So, we have $\pi_N/\pi_{N\to N-1} > 1$ as σ_s grows to ∞ for all $N \geq 3$, which means there exist a large enough $\dot{\sigma}_\epsilon$ such that $\pi_N > \pi_{N\to N-1}$ for all $\sigma_\epsilon > \dot{\sigma}_\epsilon$. The case of $N=2$ is covered in the proof of Proposition 8.

Writing π_N and π_{M} in Σ_ϵ and Σ_0 gives us:

$$
\begin{aligned}
&\lim_{\sigma_s\to\infty}\frac{\pi_N}{\pi_{\mathrm{M}}}\\
&=\lim_{\sigma_s\to\infty}\frac{\sqrt{3-4/N}}{2(N-2)}\sqrt{\frac{\Sigma_0(1-(\frac{\Sigma_\epsilon/\Sigma_0}{N-1})^{1-2/N})^2}{(1-(\frac{\Sigma_\epsilon/\Sigma_0}{N-1})^{3-4/N})(\frac{N-1}{\Sigma_\epsilon/\Sigma_0}-1)}}\Bigg/\frac{\Sigma_0}{2\sqrt{\Sigma_0+\Sigma_\epsilon}}\\
&=\frac{\sqrt{3-4/N}}{2(N-2)}\lim_{\sigma_s\to\infty}\frac{2\sqrt{N-1}(\frac{\Sigma_\epsilon/\Sigma_0}{N-1})^{1-2/N}(\frac{\Sigma_\epsilon/\Sigma_0}{N-1})^{1/2}}{(\frac{\Sigma_\epsilon/\Sigma_0}{N-1})^{3/2-2/N}}\\
&=\frac{\sqrt{(3-4/N)(N-1)}}{N-2}
\end{aligned}
$$

$\sqrt{(3-4/N)(N-1)}/(N-2)$ decrease in N and equals 1.8257 at $N=3$, 1.2245 at $N=4$, and 0.9888 at $N=5$. So, for $N=3,4$, there exists $\bar{\bar{\sigma}}_s$ such that for $\sigma_s>\bar{\bar{\sigma}}_s$, $\pi_N>\pi_{\mathrm{M}}$.

Denote by $\hat{\pi}_N$ each of N informed trader's expected profit without disclosure. The case of $N=2$ is covered in the proof of Proposition 8. Refine Σ_ϵ as Σ_ϵ/Σ_0, from equations (41) and (57) in BCW (2000) we have the following result for $N\geq 3$

$$\frac{\hat{\pi}_N}{\hat{\pi}_{\mathrm{M}}}=\frac{\sqrt{1+\Sigma_\epsilon}\int_1^\infty x^{-2/N}e^{-x\Sigma_\epsilon/N}\,dx}{N\left(\int_1^\infty x^{2(N-2)/N}e^{-2x\Sigma_\epsilon/N}\,dx\right)^{\frac{1}{2}}}$$

Applying Cauchy-Schwartz Inequality, we get

$$
\begin{aligned}
\int_1^\infty x^{-2/N}e^{-x\Sigma_\epsilon/N}\,dx&=\int_1^\infty x^{-1}x^{1-2/N}e^{-x\Sigma_\epsilon/N}\,dx\\
&\leq\left(\int_1^\infty x^{-2}\,dx\right)^{\frac{1}{2}}\left(\int_1^\infty x^{2(1-2/N)}e^{-2x\Sigma_\epsilon/N}\,dx\right)^{\frac{1}{2}}\\
&=\left(\int_1^\infty x^{2(1-2/N)}e^{-2x\Sigma_\epsilon/N}\,dx\right)^{\frac{1}{2}}
\end{aligned}
$$

On the other hand, it is clear

$$
\begin{aligned}
\int_1^\infty x^{-2/N}e^{-x\Sigma_\epsilon/N}\,dx\leq\int_1^\infty e^{-x\Sigma_\epsilon/N}\,dx=\frac{N}{\Sigma_\epsilon}e^{-\Sigma_\epsilon/N},\quad\text{and}\\
\int_1^\infty x^{2(N-2)/N}e^{-2x\Sigma_\epsilon/N}\,dx\geq\int_1^\infty e^{-2x\Sigma_\epsilon/N}\,dx=\frac{N}{2\Sigma_\epsilon}e^{-2\Sigma_\epsilon/N}
\end{aligned}
$$

Combining the above inequalities gives:

$$\frac{\hat{\pi}_N}{\hat{\pi}_{\mathrm{M}}} \leq \min\left\{\frac{\sqrt{1+\Sigma_\epsilon}}{N}, \sqrt{\frac{2(1+\Sigma_\epsilon)}{N\Sigma_\epsilon}}\right\} \leq \frac{\sqrt{1+2N}}{N} < 1. \square$$

Proof of Corollary 4 In the presence of trade disclosure, when $\sigma_s > \bar{\bar{\sigma}}_s$ and $1 < N < 5$, from Proposition 10 we have $\pi_N > \pi_{\mathrm{M}}$ which means an investor earns more when all investors acquire information and trade on the same stock than when he is the only one who acquires information and trade on that stock. So if one investor perceives other $N-1$ are acquiring information on a stock, he is better off to acquire information on that stock than to to acquire information on a different stock. $\square$