The interaction between security lending market and security trading market

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Abstract

We develop a simple model to address the interaction between security lending market and security trading market. Whether a security is hard to borrow or not results in two different scenarios of the interaction process. When a security is easy to borrow, short-selling leads to a lower spot price. When a security is hard to borrow, any CHANGE in shorting supply/demand should be largely absorbed by the lending market, and thus have minimal impact on the spot price. We identify three types of market segmentation that contribute to the observed small aggregate short interest and positive lending fee. A positive lending fee implies that the negative opinion of short sellers is offset by the opposite view of security lenders, leaving the equilibrium security price that reflects only the perception of those who neither lend nor short.

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We further perform static analyses of the change in the scope of the short-selling prohibition, the population mass of potential lenders, the degree of heterogeneity in beliefs, institutional ownership, and margin requirement. The analytical results derived from this model are potentially useful for resolving the debate on the impact of short-selling on security price.

Key words: Short-selling; Security lending; Market segmentation.
JEL classification: G12; G14

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1 Introduction

Due to regulations, short sellers have to borrow shares to fulfill trade orders. This forms a security lending market in which lenders provide shorting supply and earn lending fees. This paper establishes a simple equilibrium model to study the interaction between the security trading market and the security lending market. We distinguish between two different scenarios: (1) when the security is easy to borrow, the presence of excess shorting supply and competition among potential lenders should reduce the lending fee to zero if we ignore any brokerage transaction cost, and a lower degree of exogenous short selling constraints eventually leads to a lower spot price, which is consistent with the results of classical studies on short selling constraints; (2) when the security is hard to borrow (a.k.a. "Special"), the lending fee becomes positive under certain condition, which implies that any exogenous shock on shorting supply/demand is mostly absorbed by the lending market and has minimal

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impact on the spot price.[1] The second scenario explains a recent puzzling experimental finding by Kaplan, Moskowitz, and Sensoy (KMS hereafter) (2013) that a sizable shock in the shorting supply has no significant impact on the stock return.

Theoretically, with heterogeneous beliefs on security valuation, the security lending market may affect the spot security price in various channels. The seminal work by Miller (1977) shows that short-selling constraints exclude the opinion of pessimists and leave the stock price upward-biased, which implies low future returns. Duffe, Gâleanu, and Pedersen (DGP hereafter) (2002) provide a search model to prove that the security price may be even higher when the short-selling constraints are partially relaxed and lenders have more bargaining power to charge a high lending fee; thus, the expectation of high lending fee income may lead potential lenders to bid up the security price in the spot market; but their cases are special and limited to IPOs and firms with small float where shorting supply is very small.

More recently, assuming an given exogenous proportion of shares are lent out in the stock lending market and investors ignore the lending fee income when they decide whether to long a stock, Blocher et al. (2013) find that stocks that are hard to borrow experience higher price. In contrast, we incorporate the lending fee income into investors' valuation and endogenize the lending quantity, given the fact that only some optimistic potential lenders actually lend out their shares.

According to the data provided by MARKIT Securities Finance and Interna-

[1] Here, we consider shorting supply/demand shocks without changing fundamental information content.

tional Securities Lending Association, the global average balance of securities on loan was nearly US$2 trillion from January 2010 to January 2012, approximately US$12 trillion of lendable securities from more than 20,000 potential lenders were in the database by 2012, and European investors earned at least €1 billion of securities lending revenue during 2011. Although the security lending profit is substantial, the market size of the security lending industry is still quite small compared with the vast number of securities held worldwide. Intuitively, if there is no restriction, optimistic shareholders are always willing to lend their shares to earn incremental lending fees without changing their long position.[2] As a result, competition among potential lenders should decrease the lending fee to almost zero (their marginal cost).

Dechow et al. (2001) document that the short interest (the total shares sold short divided by the total shares outstanding) of more than 98% of NYSE/AMEX firms during the period 1976–1993 was less than 5%.[3] Chen et al. (2002) find it puzzling and raise a question: Why is there so little aggregate short interest in the real market despite the potential positive risk-adjusted return to strategies with short-selling?

We identify three types of market segmentation in short-selling and security lending that may lead to a small aggregate short interest but a positive lending fee (see Table 1). The first is on the supply side: only a portion of in-

[2] Security lenders usually cover their loan exposures with enough collateral and have the right to recall their shares at any time. For an equity loan, borrowers also have the obligation to pay dividends to the lenders if the company pays dividends during the loan period.

[3] Using Compustat data, we compute the short interest for S&P500 firms during the period 1998–2013. It is 5.47% on average across firms and months.

vestors are qualified to be security lenders, while the remaining investors (non-lenders) are prevented from lending securities. Potential lenders are usually large institutions. They hold a considerable number of shares in their inventory and pursue a passive buy-and-hold strategy so that they can provide idle lendable shares. Typical examples of potential lenders include pension funds, index funds, and insurance companies. In contrast, speculators, arbitrageurs and small investors, rarely participate in security lending. Most shareholders do not engage in security lending due to small inventory size, a short holding period, legal restrictions, voting right protection, or knowledge limitations. Typical examples of non-lenders include hedge funds, retail investors, shareholders who want to keep their voting right, and fund managers who fear that security lending will allow short sellers to push the price down.[4] Under certain condition, the scarcity of lendable shares may lead to relatively small aggregate short interest and a positive lending fee.

The second type of market segmentation is on the demand side. According to Almazan et al. (2004), more than 70% of mutual funds and almost all pension funds are prohibited from short-selling by their formal investment policy. These investors are excluded from the borrowing side of the lending market. This exogenous constraint also results in small aggregate short interest but a low lending fee. If the scope of the prohibitions is sufficiently large such that the effective demand for short-selling is too low, then the spot price

[4] In some cases, brokers may lend the shares of their clients to fulfill the small shorting orders of other clients. However, brokers play the role of security lenders and gain lending fee income in these non-substantial cases. Extant literature (e.g. DGP, 2002) usually ignores all retail investors and focus on institutional investors who are sophisticated enough to participate in the security lending market.

Table 1

Three types of market segmentation in the security lending market.

	Unrestricted	**Restricted**
Lending	Potential lenders	Non-lenders
Borrowing	Short-permitted investors	Short-banned investors
Rebate	Institutional investors	Retail investors

cannot reflect the view of pessimistic investors. This is consistent with the popular viewpoint put forward by Miller (1977) that short-selling constraints lead to upward-biased stock price. However, we show that as long as the scope of prohibition does not exceed a threshold in which the lending fee reduces to zero, this kind of exogenous short-selling constraint does not have much impact on the spot price.

The third type of market segmentation is between institutional and retail investors. When there is competition in the security lending market, lenders usually have to rebate part of the interest generated from the cash collateral (short-selling proceeds plus the extra margin) to short sellers. The rest of the interest is obtained by lenders as their lending income. However, retail investors are not eligible to receive any interest rebate because their shares are registered under the "street name" of the brokerage firm. This reduces retail investors' interest to engage in short-selling and leads to small aggregate short interest.

With these types of market segmentation, the size of the lending market should be small and a positive lending fee may arise, which implies that the short sellers' negative opinion is mostly offset by the positive opinion of lenders who incorporate the lending fee income into their valuation, leaving the spot price determined only by those who neither lend nor short. When an exogenous shock on shorting supply/demand occurs without affecting the valuation of optimistic non-lenders, the security lending market acts as a buffer to absorb the shock but has little extra impact on the spot price. In other words, the shock affects only the lending fee, which is the equilibrium price of the security lending market, but the spot price is still determined by those who do not participate in security lending and short-selling. This buffering effect increases with the seriousness of the lending-side (or supply-side) market segmentation and decreases with the seriousness of the borrowing-side (or demand-side) market segmentation. However, if the exogenous shock is too large for the security lending market to buffer, then the shock will affect the spot price.

We carry out a set of static analyses of how the security price, lending fee, and short interest are affected by each of the following exogenous driving factors in our model: (1) the scope of the short-selling prohibition, (2) the population mass of potential lenders, (3) the degree of heterogeneity in beliefs, (4) institutional ownership, and (5) the margin requirement ratio for short-selling. These analyses can potentially provide a comprehensive explanation for the mixed empirical findings regarding the relationship between the short interest or lending fee and the subsequent stock price movement documented in the literature (see, for example, Desai et al., 2002; Jones and Lamont, 2002; Cohen et al. 2007). With endogeneity, there should not be a causal relationship between these variables in the first place. We show that the apparent relationship between them varies from positive to negative or no co-movement, depending on the driving factors.

Our study contributes to the literature in three aspects. First, we provide a simple model to study the interaction between the security market and the security lending market, which adds to the existing but scarce theoretical literature; we show that the security lending market can serve as a buffer to absorb the shocks in shorting demand/supply, which is consistent with KMS's (2013) experimental findings. Second, we endogenize the short interest as well as the lending fee, and analyze the impact of several unexamined exogenous factors, e.g., the scope of the short-selling prohibition and the number of potential lenders, on the equilibrium. Third, we posit, for the first time, that lending market segmentation is the main reason why the observed lending fee is higher but the observed aggregate short interest (lending quantity) is lower than generally expected.

The remainder of this paper is organized as follows. In the next section, we survey the relevant literature. In section 3, we present the basic model. In section 4, we analyze the impact of each driving factor in the model. In section 5, we provide numerical examples. In section 6, we further extend the basic model in two aspects. In section 7, we conclude the paper. Proofs are given in Appendix A.

2 Literature review

The standard capital asset pricing model (CAPM) assumes a homogeneous investor belief that implies no short-selling activities other than for hedging or liquidity purposes, and thus, the short-selling constraints would have no impact on security prices (see Lintner, 1971). Under heterogeneous beliefs, however, some short-selling activities are information driven. Miller (1977) argues that the short-selling constraints exclude the opinion of pessimists and leave the stock price upward-biased. Jarrow (1980) shows that the asset price can be biased either upward or downward under the short-selling constraint depending on the parameters of the economy. In contrast, Diamond and Verrecchia (1986) contend that rational investors take into consideration of the effect of short-selling constraints, so there is no overpricing after they adjust their valuation. However, none examines the possible impact of the security lending market on the stock price.

DGP (2002) put forward a search model that considers the security lending fee. In their model, lenders and short sellers search for each other and bargain regarding the lending fee. When lenders are difficult to locate, they have larger bargaining power and charge a high lending fee. Since potential lenders expect lending fee income when they purchase securities, they may bid up the security price. Therefore, the security price with costly short-selling could be higher than if short-selling is completely prohibited. The relative bargaining power of the short sellers and the security lenders as well as the intensity of the matching process determine of the lending fee and affect the security price. Examples illustrate that the equilibrium security price with short-selling allowed can be higher than the equilibrium price with short-selling disallowed. However, the difficulty in locating lenders is temporary and specific in cases of IPOs and firms with small float. In common cases, as least in the US, the lending market is quite competitive and the lenders cannot enjoy a very high bargaining power. Thus, the DGP (2002) model does not fully capture the interaction between the security trading market and the security lending market in jointly determine the spot price and lending fee, especially when the security lending market is fairly competitive, which is the case at least in today's US market.

In contrast to the relatively few theoretical studies, empirical studies on security lending have been increasing since the beginning of this century. Jones and Lamont (2002) document that stocks with high lending fees experienced low future returns. D'Avolio (2002) finds that the high lending fee is explained by the high degree of heterogeneity in beliefs and argues that non-lenders who are willing to hold a stock overpriced with a high lending fee must be highly optimistic for some other reason. Geczy et al. (2002) find that the feasibility and profitability of strategies that involve short-selling rely on the lending market, and that the IPOs, DotCom, large-cap, growth, and low-momentum stocks are cheap to borrow, but the acquirers' stocks in M&A events are expensive to borrow. Desai et al. (2002) and Asquith et al. (2005) document the negative predictive power of short interest on future returns.

3 Basic model

3.1 Model Setup

Our model consists of a risky security with a fixed number of shares, S, outstanding, two dates, and two types of investors in the market. Potential lenders, with a population mass, M, may or may not lend their shares to short sellers when they take a long position. Another segment of investors, namely, non-lenders, with a population mass, N, cannot lend their shares.

In some circumstances, both types of investors may be subject to short-selling prohibitions. Let a $(0 \leq a \leq 1)$ be the proportion of investors who are banned from short-selling. Thus, the total population mass of potential short sellers is given by $K = (1 - a)(M + N)$.

At the beginning time t_0, these two types of investors have heterogeneous expectations of the ending value of the risky security. A representative potential lender i's valuation is V_i, and a representative non-lender j's valuation is V_j. Following Chen et al. (2002), we assume that all investors' valuations are uniformly distributed on the interval $[F-cF, F+cF]$ with mean F and $c \in (0,1)$. Thus, c captures the dispersion of opinion among all investors and measures the degree of heterogeneity in beliefs in the economy. If investors have correct valuation on average, F equals the fundamental value of the security.

The optimists among potential lenders take a long position in the spot market at price p and may wish to lend their shares to earn an extra lending fee. The pessimists among all investors, who are allowed to short the security, become borrowers. To do so, the short sellers must pledge cash collateral that exceeds the short-selling proceeds by a margin requirement ratio m. The lenders keep the interest generated by the cash collateral, but they rebate part of the interest to the borrowers at a rebate rate $b \in (-\infty, r]$, where r is the fixed risk-free interest rate. The difference between r and b, which is kept by the lenders, contributes to the lending fee. A negative b means a lending fee higher than the risk-free interest rate. At the ending time t_1, the value of the security is realized. The short sellers must buy back their borrowed shares to cover their short position.

3.2 Potential lender

The representative potential lender i with initial wealth W_i^0 allocates B on the risk-free asset and the rest on q_i shares of the risky security, where a negative q_i stands for a short position.

When she takes a long position, pq_i is paid to purchase the risky security, and she can, in turn, lend it to short sellers to get $(1+m)pq_i$ cash collateral (including the short-selling proceeds) for reinvestment. Unlike non-lenders who stay away from the security lending market, a potential lender always lends out her shares to earn a positive lending fee when she takes long position. The time t_0 budget becomes

$$W_i^0 + (1+m)\,pq_i = B + pq_i. \tag{1}$$

At time t_1, she returns the collateral, rebates part of the interest to the short sellers at rate b, and claims back her shares. Her expected ending wealth at time t_1 is given by

$$W_i^1 = (1+r)\,B - (1+b)\,(1+m)\,pq_i + q_i V_i \tag{2}$$

When she takes a short position, she should put $mp|q_i|$ as extra collateral for the borrowed shares. The initial budget becomes

$$W_i^0 = B + mp|q_i|. \tag{3}$$

At time t_1, she needs to purchase the security to cover her short position and get back the collateral (including the short-selling proceeds) as well as the rebated interest. Her expected ending wealth is given by

$$W_i^1 = (1+r)B - |q_i|V_i + (1+b)(1+m)p|q_i| \tag{4}$$

Here, we assume for simplicity that all investors have zero initial wealth, but they can borrow money at a risk-free interest rate without limits. Each investor adds the dollar lending fee per share $L \triangleq p(1+m)(r-b)$ to their valuation. According to equation (2) and (4), her expected ending wealth is given by

$$W_i^1 = q_i[V_i - p(1+r) + L]. \tag{5}$$

We also assume for simplicity that all investors have the same constant risk aversion coefficient A and the same valuation variance σ^2. Using a standard mean-variance utility function for the risk-averse investor, we solve the problem:

$$\max U_i = E(W_i^1) - \frac{A}{2}Var(W_i^1) = q_i[V_i - p(1+r) + L)] - \frac{A}{2}q_i^2\sigma^2. \tag{6}$$

From the first-order condition, investor i's optimal demand for the security is given by

$$q_i = \lambda[V_i - p(1+r) + L], \tag{7}$$

where $\lambda \triangleq \frac{1}{A\sigma^2}$ is the investor's propensity to risk. When her valuation of the risky security V_i plus the lending fee income L exceeds the opportunity cost $p(1+r)$, she takes a long position and becomes a security lender; otherwise, she becomes a short seller.

3.3 Non-lender

A representative non-lender j cannot lend when she takes a long position. Similar to equation (5) except the lending fee income, her ending wealth is given by

$$W_j^1 = \begin{cases} q_j \left[V_j - p\left(1+r\right)\right], & \text{if } q_j \geqslant 0, \\ \\ q_j[V_j - p\left(1+r\right) + L], & \text{if } q_j < 0. \end{cases} \tag{8}$$

Similarly, her demand of the security is given by

$$q_j = \begin{cases} \lambda[V_j - p(1+r)], & \text{if } V_j > p(1+r), \\ \\ 0, & \text{if } p(1+r) - L \leqslant V_j \leqslant p\left(1+r\right), \\ \\ \lambda[V_j - p(1+r) + L)], & \text{if } V_j < p(1+r) - L. \end{cases} \tag{9}$$

Therefore, she may take a long position, no position, or a short position depending on her personal assessment, taking as given the security price and the lending fee.

3.4 The equilibrium

The equilibrium is given by a pair $\{p^*, L^*\}$ such that the spot market and the lending market clear simultaneously.

From equation (7), the aggregate demand of potential lenders is given by

$$Q_I = \frac{M}{2cF}\left\{ \int\limits_{p(1+r)-L}^{F+cF} \lambda[V_i - p(1+r) + L]dV_i + (1-a) \int\limits_{F-cF}^{p(1+r)-L} \lambda[V_i - p(1+r) + L]dV_i \right\}. \tag{10}$$

Similarly, the aggregate demand of non-lenders is given by

$$Q_J = \frac{N}{2cF}\left\{ \int\limits_{p(1+r)}^{F+cF} \lambda[V_j - p(1+r)]dV_j + (1-a) \int\limits_{F-cF}^{p_0(1+r)-L} \lambda[V_j - p\,(1+r) + L]dV_j \right\}. \tag{11}$$

In equilibrium, the total demand must equal the number of total shares outstanding, that is

$$Q_I + Q_J = S. \tag{12}$$

In the lending market, the shorting supply comes from the long position of optimistic potential lenders, which is given by

$$SS = \frac{M}{2cF} \int\limits_{p(1+r)-L}^{F+cF} \lambda[V_i - p(1+r) + L]dV_i. \tag{13}$$

The aggregated shorting demand, which comes from short sellers in both groups, is given by

$$SD = -\frac{K}{2cF} \int\limits_{F-cF}^{p(1+r)-L} \lambda[V_s - p(1+r) + L]dV_s. \tag{14}$$

As illustrated below, the following condition ensures that the equilibrium lending fee is non-negative ($L^* \geqslant 0$):

Condition A $\frac{M}{K} \leqslant \left(\sqrt{\frac{\lambda NcF}{S}} - 1\right)^2$.

The left side is the population mass ratio of the potential lenders to the potential short sellers. Obviously, Condition A tends to be satisfied when the market segmentation on the lending side is strong (M is small), when the market segmentation on the borrowing side is weak (K is large), or when the degree of heterogeneity in beliefs (c) is high. To compute the equilibrium outcome, we distinguish two scenarios:

Scenario A: When Condition A is satisfied, the market segmentation is binding on the lending side. A positive lending fee arises from the limited shorting supply, suggesting that the security may be "special" and relatively hard to borrow. The market clearing conditions reduce to the following two sets of equality constraints for the spot market and the lending market, respectively:

$$\begin{cases} \frac{N}{2cF} \int\limits_{p(1+r)}^{F+cF} \lambda[V_j - p(1+r)]dV_j = S \\ SS = SD \end{cases}. \tag{15}$$

The first equation suggests that the aggregated demand from optimistic non-lenders must coincide with the number of shares outstanding. The second equation suggests that optimistic lenders' total share holdings offset the aggregated demand for security lending from all short sellers. From the two equations, we can readily ascertain the equilibrium $\{p^*, L^*\}$ along with the equilibrium short interest SI^* that is defined as the total short quantity divided by the number of shares outstanding.

The logic in Scenario A is very simple. Given a positive lending fee, the security lending market must be clear; otherwise, rational marginal security lenders are always willing to lend their shares to earn incremental lending fee income since their opportunity cost to lend is almost zero (only the transaction cost). That is to say, all lendable shares in inventory should have been lent out and the rest are un-lendable for some alternative reason. To facilitate any new shorting demand, the new shorting supply must be created from a new long position that just offsets the short position in the spot market. In equilibrium, the lending fee increases, but the spot price remains unchanged. Similarly, when a shorting supply shock occurs, the lending fee rather than the spot price is affected. [5]

Scenario B: When Condition A is violated, the market segmentation is binding on the borrowing side. There exists excess shorting supply (see Figure 1), and the competition reduces the lending fee to zero, implying that the security is easy to borrow. Since, in this case, security lenders are indifferent between lending or not lending, the market clearing condition in the spot market will determine the equilibrium security price. Given the equilibrium security price and the zero lending fee, the short interest will be fully determined by the aggregated shorting demand. Precisely, setting

$$S = \frac{M}{2cF}\int_{p(1+r)}^{F+cF} \lambda[V_i - p(1+r)]dV_i + \frac{N}{2cF}\int_{p(1+r)}^{F+cF} \lambda[V_j - p(1+r)]dV_j \tag{16}$$
$$+\frac{K}{2cF}\int_{F-cF}^{p(1+r)} \lambda[V_s - p(1+r)]dV_s$$

[5] Even if the market friction in the lending market is so significant that not all shares are lent out, similar market friction in the spot market also dampens the effect (if any) of the shorting demand/supply shocks on the spot price.

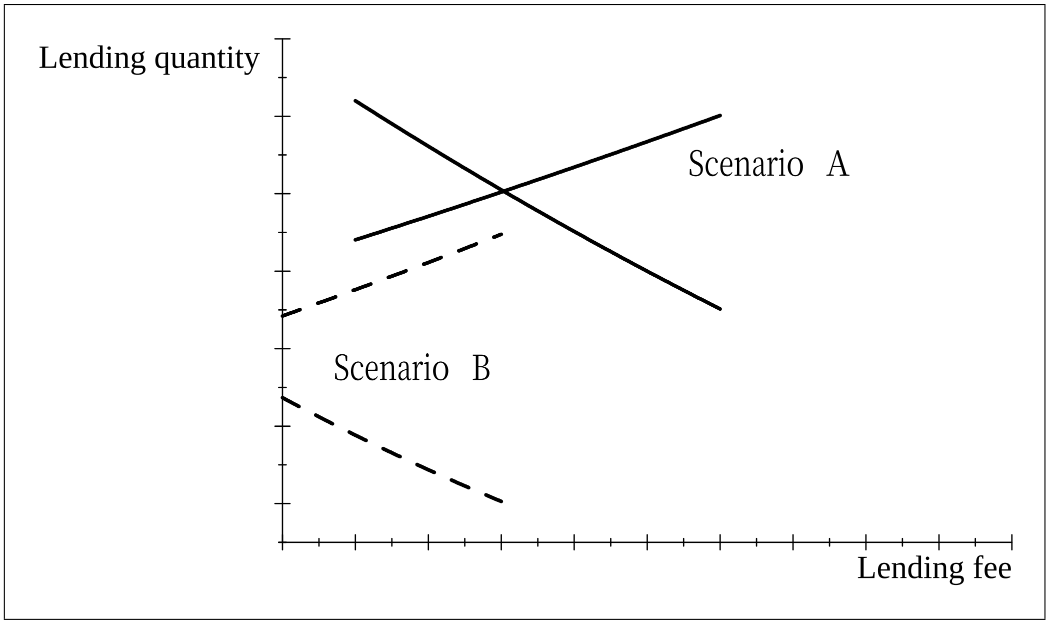

Fig. 1. The equlibrium in the security lending market

We can also solve for the equilibrium security price along with the short interest.

Combining these two scenarios together and putting them formally, we have:

Theorem 1 *For any equilibrium in Scenario A, the supply from lenders always offsets shorting demands from all short sellers, so the equilibrium security price is determined only by the aggregated demand from the optimistic non-lenders who choose to take some long positions. In this case, the security price, lending fee, and short interest admit the following analytic expressions:*

$$p^* = \frac{1}{1+r}\left(F + cF - 2\sqrt{\frac{cSF}{\lambda N}}\right) \tag{17}$$

$$L^* = 2\left(\frac{\sqrt{K}}{\sqrt{K}+\sqrt{M}}cF - \sqrt{\frac{cSF}{\lambda N}}\right) \tag{18}$$

$$SI^* = \frac{\lambda M cF}{S}\left(\frac{\sqrt{K}}{\sqrt{K}+\sqrt{M}}\right)^2 \tag{19}$$

For any equilibrium in Scenario B, the lending fee remains at zero ($L^* =$

0). The security price and the short interest admit the following analytic expressions:

$$p^* = \begin{cases} \frac{1}{1+r}\left[F + \frac{(M+N+K)cF - 2\sqrt{K(M+N)c^2F^2 + (M+N-K)\frac{cSF}{\lambda}}}{M+N-K}\right], & \text{if } 0 < a \leqslant 1, \\ \frac{1}{1+r}\left[F - \frac{S}{\lambda(M+N)}\right], & \text{if } a = 0. \end{cases} \tag{20}$$

$$SI^* = \begin{cases} \frac{\lambda K}{4cFS}\left[\frac{2(M+N)cF - 2\sqrt{K(M+N)c^2F^2 + (M+N-K)\frac{cSF}{\lambda}}}{M+N-K}\right]^2, & \text{if } 0 < a \leqslant 1, \\ \frac{\lambda(M+N)}{4cFS}\left[cF - \frac{S}{\lambda(M+N)}\right]^2, & \text{if } a = 0. \end{cases} \tag{21}$$

Particularly, when there is neither an exogenous short-selling prohibition ($a = 0$) nor an endogenous short-selling cost ($L = 0$), we obtain the security price given by the CAPM under heterogeneous beliefs: [6]

$$p^{\text{CAPM}} = \frac{1}{1+r}\left[F - \frac{S}{\lambda(M+N)}\right]. \tag{22}$$

4 Static analyses

4.1 Scope of the short-selling prohibition

The scope of the short-selling prohibition, a, measures the borrowing-side market segmentation. According to Theorem 5, its impact on the equilibrium outcome is mixed.

Proposition 1 *In Scenario A, as the scope of the short-selling prohibition (a)*

[6] See Sun and Yang (2003) for more details about the CAPM under heterogeneous beliefs.

increases, the security price remains unchanged; the lending fee and the short interest decrease. In Scenario B, as a increases, the security price increases, the lending fee remains unchanged, and the short interest decreases.

All proofs are given in Appendix A.

Scenario A prescribed in Proposition 1 is inconsistent with the popular viewpoint that short-selling constraints result in overpricing. Its validity requires an intuitive explanation. The exogenous short-selling prohibitions have no impact on the security price when the scope of the prohibition is not large enough. In this case, although some negative opinion is excluded by the short-selling prohibition, the drop in shorting demand reduces the lending fee and lowers the valuation of potential lenders, to offset the excluded negative sentiment from the short sellers. Therefore, the impact of the tightened short-selling constraints by an external force is absorbed through the shrinkage of the lending market.

We can also provide an intuitive explanation for the statement made in Scenario B of Proposition 1. Indeed, when the coverage of the short-selling prohibition exceeds the absorbing ability of the lending market, the profitability of security lending is removed by the decrease in shorting demand. As a result, the excluded negative opinion by the short-selling constraints must be reflected in the upward-biased security price.

In particular, when the short-selling prohibition applies to all investors ($a = 1$), as was the case when short-selling for financial stocks were banned during the financial turmoil in 2008, it yields an equilibrium price (in Scenario B) in which

$$p^{\text{ban}} = \frac{1}{1+r}\left[F + cF - 2\sqrt{\frac{cSF}{\lambda(M+N)}}\right]. \tag{23}$$

Compared with the case in Scenario A in which all investors are permitted to short ($a = 0$), the net price effect is given by

$$p^{\text{ban}} - p^{*} = \frac{2}{1+r}\sqrt{\frac{cSF}{\lambda}}\left(\sqrt{\frac{1}{N}} - \sqrt{\frac{1}{M+N}}\right) > 0. \tag{24}$$

Obviously, the impact of the short-selling ban not only is determined by the divergence of opinion but is also related to the population mass of potential lenders. The up-bias effect of the short-selling ban on the security price is stronger when there are more potential lenders.

4.2 Population mass of potential lenders

The population mass of potential lenders, M, measures the lending-side market segmentation. A large number of potential lenders can mitigate the market segmentation.

Proposition 2 *In Scenario A, as the population mass of potential lenders (M) increases, the security price remains unchanged; the lending fee decreases; the short interest increases. In Scenario B, as M increases, the security price increases, the lending fee remains unchanged, and the short interest increases.*

In Scenario A, when more potential lenders enter the market, they contribute more to the shorting supply (from optimists among them who are TRUE lenders) than to the shorting demand (from pessimists among them who are short sellers), because these new potential lenders are only faced with market segmentation on borrowing side. The lower lending fee is attributable to the net incremental shorting supply. The larger shorting volume leads to the higher short interest, but the security price remains intact as long as the lending fee is still positive, because the new true lenders and new short sellers must offset each other as long as the lending market is clear. In this case, the security price should not co-move with the short interest or the lending fee.

When more and more potential lenders enter the market (M is so large that Condition A is violated), they may bring excess shorting supply and shift the scenario from A to B. In such cases, the lending market is not clear and the lending fee remains at zero with the increasing short interest. The excess new shareholders who cannot be offset by new short sellers elevate the spot price in the security trading market.

4.3 Heterogeneity in beliefs

Scheinkman and Xiong (2003) establish a model that heterogeneous beliefs arise from over-confidence. Hong and Stein (2003) show how short-selling activities lead to a market crash under heterogeneous beliefs. Our model provides a simple approach for investigating the impact of heterogeneous beliefs when the security lending activity is endogenized.

Proposition 3 *In Scenario A, as the heterogeneity in beliefs (c) increases, the security price, lending fee, and short interest increase. In Scenario B, as c increases, the security price may decrease or increase, the lending fee remains unchanged, and the short interest increases.*

When investors' opinion diverges, shorting supply from optimists and shorting demand from pessimists shifts upward, so the short volume always increases. In Scenario A, because the lending power is limited by market segmentation, the shorting demand increases more than the shorting supply. The net effect on the lending fee should be positive. Meanwhile, since the opinion of optimistic non-lenders is strengthened at the spot market, the equilibrium price is lifted upward as well. In Scenario B, however, since the spot price is determined by all investors, the net effect on the price can be bidirectional.

4.4 Summary of static analyses

The impact of an increase in each driving factor on the security price, lending fee, and short interest in two scenarios are summarized in Table 2. The relationship between the lending fee and the stock price could be positive or NIL and the relationship between the short interest and the stock price could be positive, negative or NIL, depending on the scenarios and driving factors.

Extant empirical studies focus on securities with a significant positive lending fee (Scenario A). For example, Cohen et al. (2007) observe the shift of shorting demand/supply curve, and document 2-3% negative monthly abnormal returns after positive shocks in shorting demand, but they find little evidence of a spot price effect after shocks in the shorting supply. We have shown that shorting demand and shorting supply are simultaneously driven by specific factors. An increase in the divergence of opinion leads to an increase in the shorting demand, which may predict a lower future return. However, a decrease in the population mass of potential lenders reduces the short supply, which may have no impact on the future return.

Table 2

Impact of driving factors on the equilibrium outcomes.

Driving factors	Security price	Lending fee	Short interest
Scenario A: the lending fee is positive			
Scope of short-selling prohibition	NIL	Negative	Negative
Population mass of potential lenders	NIL	Negative	Positive
Heterogeneity in beliefs	Positive	Positive	Positive
Scenario B: the lending fee is zero			
Scope of short-selling prohibition	Positive	NIL	Negative
Population mass of potential lenders	Positive	NIL	Positive
Heterogeneity in beliefs	Mixed	NIL	Positive

5 Numerical examples

Suppose an economy with $\lambda = 1$; $S = N = 10000$; $F = 10$; $r = 0.03$. In each numerical exercise, we let one of the model parameters vary, and keep the other model parameters unchanged. For each parametric assignment we compute the equilibrium outcome, including the lending fee (L), the security price (p), and the short interest (SI). If we assume that the investors have correct valuation on average, we can calculate the future gross return of the security with

$$R = \frac{F}{p}. \tag{25}$$

As a benchmark, the security price and future return implied by CAPM is $p^{CAPM} = 8.9$ and $R^{CAPM} = 1.124$ (12.4% net return) respectively.

Table 3

Change in the equilibrium outcome with the increase in the scope of the short-selling prohibition when c=0.3 and M=2000.

a	p	L	SI	R
0	9.258	0.796	0.303	1.080
0.2	9.258	0.656	0.283	1.080
0.4	9.258	0.465	0.257	1.080
0.6	9.258	0.182	0.222	1.080
0.8	9.346	0	0.138	1.070
1	9.551	0	0	1.047

Table 3 and Figure 2 illustrate the effect of an exogenous short-selling ban on the security market. We see that unless the scope of the short-selling ban (a) is sufficiently large, it has no impact on the spot price, but decreases the lending fee and the short interest because of the lower shorting demand. However, when a model parameter changes dramatically, it may shift the scenario from one to the other.

Table 4 and Figure 3 illustrate how the population mass of potential lenders (M) affects the equilibrium outcome. We see that, as more potential lenders enter the market, the lending fee drops, and the short interest rises with the higher shorting supply. However, it has no impact on the security price unless M reaches such a high level that the lending fee reduces to zero.

Table 5 and Figure 4 illustrate the effect of dispersion in beliefs (c) on the equilibrium outcome. The spot price, lending fee, and short interest all increase

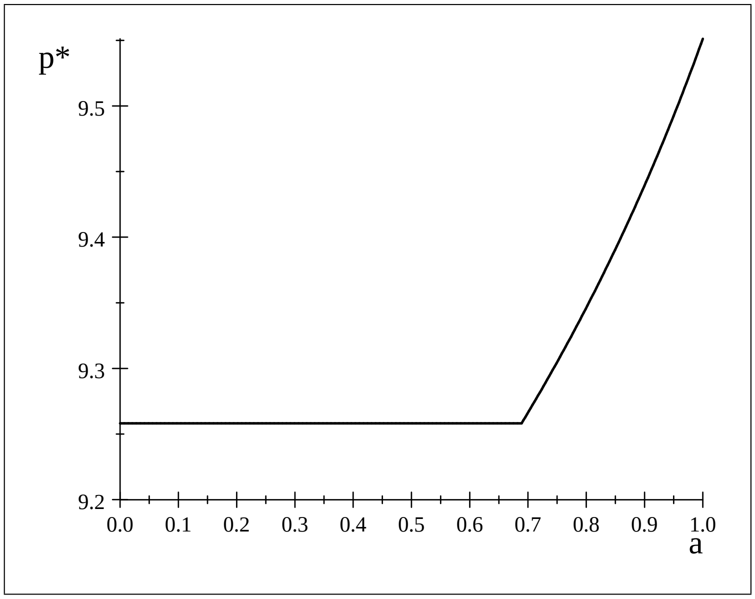

Fig. 2. The equilibrium spot price with different scopes of the short-selling prohibition.

Table 4

Change in the equilibrium outcome as the population mass of potential lenders increases when a=0.5 and c=0.3.

M	p	L	SI	R
500	9.258	1.192	0.090	1.080
1000	9.258	0.797	0.151	1.080
2000	9.258	0.340	0.241	1.080
3000	9.258	0.051	0.309	1.080
4000	9.330	0	0.341	1.072
5000	9.697	0	0.364	1.031

with the diverging of opinion when it is sufficiently large.

In each table, the lending fee is positive when Condition A is satisfied. In

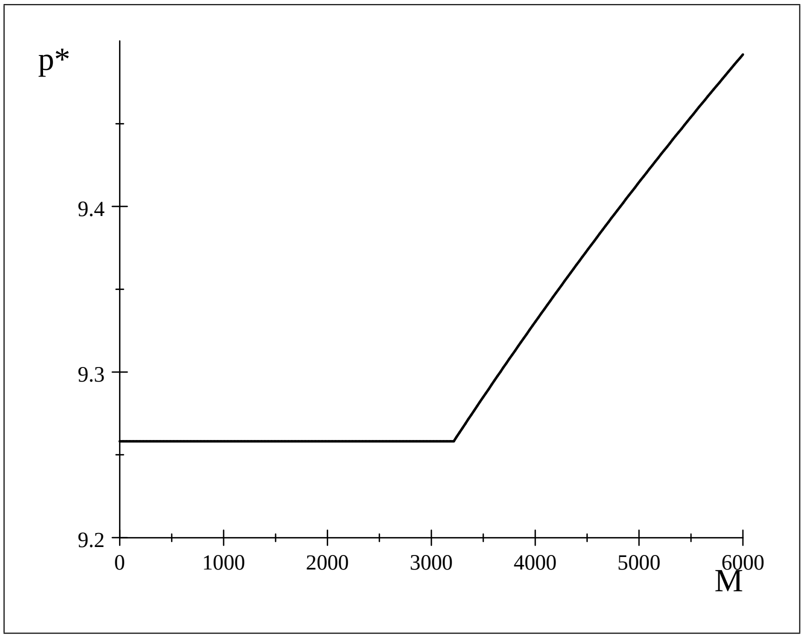

Fig. 3. The equilibrium spot price with the evolution of the population mass of potential lenders.

Table 5

Change in the equilibrium outcome with the increase in heterogeneity in beliefs when a=0.5 and M=2000.

c	p	L	SI	R
0.1	8.903	0	0.004	1.123
0.2	8.997	0	0.120	1.111
0.3	9.258	0.340	0.241	1.080
0.4	9.709	1.072	0.322	1.030
0.5	10.221	1.868	0.402	0.978
0.6	10.778	2.709	0.482	0.928

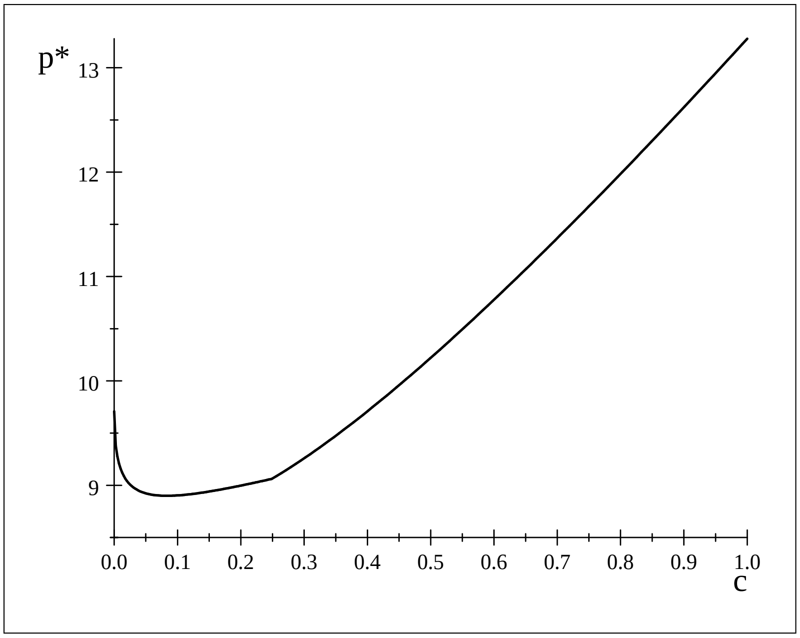

Fig. 4. The equilibrium spot price with the evolution of the heterogeneity in beliefs.

this circumstance, the spot price is affected only by c, and all three factors simultaneously affect the lending fee and short interest. In contrast, when Condition A is violated, the lending fee remains at zero. In this case, all three factors affect the spot price and short interest but have no impact on the lending fee.

6 Model extensions

Here, we extend the model in two aspects: First, we introduce retail investors into the model. The retail investors are discriminated against when they conduct short-selling transactions because these investors have to pay a higher fee than institutional investors. Second, we consider the possibility that investors are subject to budget constraints. For simplicity, we assume that there is no short-selling prohibition ($a = 0$) and that the condition for non-negative lending fee is satisfied.

6.1 The role of retail investors

The third dimension of market segmentation exists between institutional investors and retail investors. Retail short sellers cannot receive a positive interest rebate, and usually face higher margin requirements than institutional short sellers. This leads to a higher lending fee that is given by

$$L^{\text{retail}} \triangleq p\left(1 + m^{\text{retail}}\right)(r - b^{\text{retail}}), \tag{26}$$

where $m^{\text{retail}} > m$ and

$$b^{\text{retail}} = \begin{cases} 0 \text{ if } b \geqslant 0, \\ \\ b \text{ if } b < 0. \end{cases} \tag{27}$$

According to Theorem 5, all the S shares are held by non-lenders.[7] Among them, let θ be institutional ownership and the rest be the ownership of retail investors. The lending market clearing condition becomes

$$\begin{aligned} 0 = {} & \frac{M}{2cF} \int_{F-cF}^{F+cF} \lambda[V_i - p(1+r) + L] dV_i \\ & + \frac{N(1-\theta)}{2cF} \int_{F-cF}^{p(1+r)-L^{\text{retail}}} \lambda[V_j - p(1+r) + L^{\text{retail}}] dV_j \\ & + \frac{N\theta}{2cF} \int_{F-cF}^{p(1+r)-L} \lambda[V_j - p(1+r) + L] dV_j \end{aligned} \tag{28}$$

Since retail investors act in the same way as institutional non-lenders in the long side, the equilibrium security price will not be affected when θ changes.

[7] Economically, the lender also takes a long position. Legally, however, the title of the shares is under the names of the buyers of the short orders.

However, because retail investors face higher short-selling costs, the shorting demand of pessimistic retail investors is lower than that of institutional investors with the same valuation. Therefore, the total shorting demand increases when θ increases. This, in turn, leads to higher equilibrium lending fees (for all short sellers) and higher short interest. Thus, we immediately obtain

Proposition 4 *As institutional ownership (θ) increases, the security price remains unchanged; the lending fee for all investors increases, and the short interest increases.*

In the empirical literature, most people treat institutional ownership as a proxy for shorting supply; see, for instance, Nagel (2005) and Asquith et al. (2005). However, they do not distinguish institutional non-lenders from potential lenders. In fact, institutional ownership affects the demand side of the lending market as well as the supply side. High institutional ownership may elevate the shorting cost rather than relax the short-selling constraints.

6.2 The impact of budget constraint

Investors always face budget constraints that may restrict demand. In this section, instead of unlimited borrowing, we assume that all investors face the same budget constraint G.

Lenders can take advantage of excess liquidity from cash collateral for reinvestment so that they are not limited by the budget constraint. In contrast, as long as the margin requirement ratio, m, is larger than zero, the lower bound of the demand for a pessimistic investor (i.e. the largest amount she can short) must satisfy

$$\lambda[V_L - p(1+r) + L] = -\frac{G}{mp}, \tag{29}$$

thus, the valuation of the marginal short seller, V_L, is given by

$$V_L = p(1+r) - L - \frac{G}{\lambda mp}. \tag{30}$$

In this paper, we focus on the margin of short-selling and ignore the margin purchase. The upper bound of the demand for a optimistic non-lender must satisfy

$$\lambda[V_U - p(1+r)] = \frac{G}{p}, \tag{31}$$

therefore, the valuation of the marginal investor, V_U, is given by

$$V_U = p(1+r) + \frac{G}{\lambda p}. \tag{32}$$

To investigate the impact on budget constraints, we assume that the lower bound and the upper bound are binding at least on some investors, which requires $V_U < F + cF$ and $V_L > F - cF$. From the spot market clearing condition

$$\frac{N}{2cF}\left\{\int_{p_0(1+r)}^{V_U} \lambda[V_j - p(1+r)]dV_j + \frac{G}{p_0}\left[F + cF - V_U\right]\right\} = S, \tag{33}$$

we obtain the following equilibrium security price:

$$p^{\text{budget}} = \frac{(F + cF) + \sqrt{(F + cF)^2 - \frac{4cFS + 2(1+r)NG}{\lambda N}}}{\frac{4cFS}{NG} + 2(1 + r)}. \tag{34}$$

From the lending market clearing condition

$$\begin{aligned} 0 = & \frac{M}{2cF}\left\{\int_{V_L}^{F+cF} \lambda[V_i - p(1 + r) + L]dV_i - \frac{G}{mp}[V_L - (F - cF)]\right\} \\ & + \frac{N}{2cF}\left\{\int_{V_L}^{p(1+r)-L} \lambda[V_j - p(1 + r) + L]dV_j - \frac{G}{mp}[V_L - (F - cF)]\right\}, \end{aligned} \tag{35}$$

we solve for the equilibrium lending fee:

$$\begin{aligned} L^{\text{budget}} = & p^{\text{budget}}(1 + r) \\ & + \frac{\sqrt{(M + N)(G^2 + 4cF\lambda M m p^{\text{budget}} G)} - (M + N)G}{\lambda M m p^{\text{budget}}} \\ & -(F + cF), \end{aligned} \tag{36}$$

and the short interest is given by

$$SI^{\text{budget}} = \frac{\lambda M}{4cF}\left[\frac{\sqrt{(M + N)(G^2 + 4cF\lambda M m p^{\text{budget}} G)} - (M + N)G}{\lambda M m p^{\text{budget}}}\right]^2. \tag{37}$$

The impact of G is quite complicated. Since the security price is determined only by the optimists in non-lenders, relaxing the constraint may enable them to buy more shares, which drives up the security price. However, a higher price may adversely tighten the budget constraints. The valuation of the marginal investor decreases because the budget constraint is binding for more investors. As a result, the security price could be lower because the aggregate demand is lower, although the budget constraint is looser.

Recall the numerical example with $\lambda = 1$; $S = N = 10000$; $F = 10$; $r = 0.03$;

$c = 0.3$; $M = 2000$. If the budget constraint (G) is 20, the equilibrium security price is 8.933. The price increases to 9.251 with the budget of 30, and falls back to 9.168 with the budget of 40. We can see the non-monotonic price effect from the budget constraint change.

The margin ratio has no effect on the spot price.[8] However, the margin ratio plays a very important role in the return and risk of security lending.

Proposition 5 *Under budget constraints, as the margin requirement ratio (m) increases, the security price remains unchanged; the lending fee and the short interest increase before reaching m^* and decrease after reaching m^*, where*

$$m^* = \frac{\left(\sqrt{M} + \sqrt{M+N}\right) G}{2cF\lambda p^{budget}\sqrt{M}}$$

On one hand, a higher margin requirement ratio improves the lending fee income because of the larger reinvestment of the collateral. As a result, short interest is also higher because lenders are more willing to lend. On the other hand, a higher margin requirement ratio reduces shorting demand because the budget constraint for short sellers is tightened. Consequently, the lending fee and the short interest decrease. A level of m exists that lenders can maximize their lending fee income.

Moreover, lenders bear the default risk when short sellers fail to cover their position when the margin is not enough to cover the loss. This may happen when the security price rises much faster than the margin adjustment. An efficient way to mitigate the default risk is to set a sufficiently high margin

[8] If margin purchases are permitted, the margin requirement ratio may affect the spot price.

requirement ratio. Lenders should choose an appropriate margin requirement ratio based on the risk and return trade-off.

7 Conclusion

With heterogeneous beliefs and various restrictions in the security lending market, we establish a model to study how short-selling activities would affect the security price. Our model provides a simple framework for exploring the extent to which and the channel in which the security lending market interacts with the spot market, and shows how both markets jointly determine the security price and the lending fee. The interaction process varies significantly between the securities that are hard to borrow (Scenario A) and securities that are easy to borrow (Scenario B).

In Scenario A, with the three types of market segmentation, the size of the lending market should be small, and a positive lending fee may arise. A positive lending fee implies that the negative opinion of short sellers is mostly offset by the positive opinion of lenders who incorporate the lending fee income into their valuation, leaving the security price determined only by investors who do not participate in security lending and short-selling. Thus, the lending market acts as a buffer that absorbs the impact from exogenous shock on shorting supply/demand and leaves the spot price the same as if there were no shock. The buffering effect is stronger when the degree of market segmentation is higher on the lending side and lower on the borrowing side.

In Scenario B, the presence of excess shorting supply and competition among potential lenders should reduce the lending fee to zero if we ignore any brokerage transaction cost, and a lower degree of exogenous short selling constraints eventually leads to a lower spot price, which is consistent with the results of classical studies on short selling constraints.

Our model suggests that the spot security price is simultaneously determined with the lending fee and the short interest. There are co-movements but not necessarily causal relationships between the short interest or the security lending fee and the subsequent spot market security price change. In fact, the relationships vary with the different driving factors. This implication is potentially useful for explaining the inconclusive empirical findings about the relationships between the short interest, the lending fee, and the security price in extant literature.

A Appendix. Proofs

A.1 Proof of Proposition 1

Denote that

$$X = \frac{\sqrt{K}}{\sqrt{K} + \sqrt{M}}. \tag{A.1}$$

By definition, $K \triangleq (1-a)(M+N)$, which decreases in a. So, X decreases in a as well.

In Scenario A, because both SI^* and L^* increase in X, they must decrease in a.

In Scenario B, the following equation holds for the equilibrium price p^*:

$$\frac{\lambda K}{4cF}\left[p^*(1+r) - (F - cF)\right]^2 + S = \frac{\lambda(M+N)}{4cF}[(F+cF) - p^*(1+r)]^2. \tag{A.2}$$

Taking derivative w.r.t. a on both sides of the equation, we obtain

$$
\begin{aligned}
& \underbrace{K'(a)}_{-} \frac{\lambda}{4cF} \left[p^*(1+r) - (F - cF)\right]^2 \\
& + \frac{\lambda K}{2cF} \underbrace{\left[p^*(1+r) - (F - cF)\right]}_{+} (1+r) \frac{\partial p^*}{\partial a} \\
= & -\frac{\lambda(M+N)}{2cF} \underbrace{\left[(F + cF) - p^*(1+r)\right]}_{+} (1+r) \frac{\partial p^*}{\partial a} \\
\Longrightarrow & \frac{\partial p^*}{\partial a} > 0.
\end{aligned}
$$

Consequently, the r.h.s. of the equation must decreases in a. This, in turn, implies the short volume, which is given by the first term on the left side of the equation, must decreases in a as well. This enables us to conclude that SI^* decreases in a.

A.2 Proof of Proposition 2

In Scenario A, p^* is invariant in M. The intersection of shorting demand and shorting supply curves yields the following equation for the lending fee L^*:

$$
\frac{\lambda K}{4cF} \left[p^*(1+r) - L^* - (F - cF)\right]^2 = \frac{\lambda M}{4cF} \left[(F + cF) - p^*(1+r) + L^*\right]^2. \quad \text{(A.3)}
$$

Taking derivative w.r.t. M on both sides of the equality, we obtain

$$
\begin{aligned}
& -\frac{\lambda K}{2cF}\underbrace{[p^*(1+r) - L^* - (F - cF)]}_{+}\frac{\partial L^*}{\partial M} \\
= \; & \frac{\lambda}{4cF}[(F + cF) - p^*(1+r) + L^*]^2 \\
& +\frac{\lambda M}{2cF}\underbrace{[(F + cF) - p^*(1+r) + L^*]}_{+}\frac{\partial L^*}{\partial M} \\
\Longrightarrow & \frac{\partial L^*}{\partial M} < 0.
\end{aligned}
$$

With this we assert that the short interest SI^*, which is given by the left hand side of the equation, increases in M as well.

In Scenario B, the market clearing condition yields the following equation for the equilibrium price p^*:

$$
\frac{\lambda K}{4cF}\left[p^*(1+r) - (F - cF)\right]^2 + S = \frac{\lambda(M+N)}{4cF}[(F + cF) - p^*(1+r)]^2. \quad \text{(A.4)}
$$

Again, taking derivative w.r.t. M on both sides of the equality, we obtain

$$
\begin{aligned}
& \frac{\lambda K}{2cF}\underbrace{[p^*(1+r) - (F - cF)]}_{+}(1+r)\frac{\partial p^*}{\partial M} \\
= \; & \frac{\lambda}{4cF}[(F + cF) - p^*(1+r)]^2 \\
& -\frac{\lambda(M+N)}{2cF}\underbrace{[(F + cF) - p^*(1+r)]}_{+}(1+r)\frac{\partial p^*}{\partial M} \\
\Longrightarrow & \frac{\partial p^*}{\partial M} > 0
\end{aligned}
$$

This, in turn, implies that SI^* increases in M.

A.3 Proof of Proposition 3

In Scenario A, taking the derivative of SI^* with respect to c, it is easy to show that the derivative is always positive.

$$\frac{\partial SI^*}{\partial c} = \frac{\lambda MFX^2}{S} > 0. \tag{A.5}$$

From Condition A, we have

$$\sqrt{\frac{S}{\lambda NcF}} \leqslant X < 1, \tag{A.6}$$

which implies $\sqrt{F} > \sqrt{\frac{S}{\lambda Nc}}$. Taking the derivative of p^* with respect to c, we obtain

$$\frac{\partial p^*}{\partial c} = \frac{\sqrt{F}}{1+r}\left(\sqrt{F} - \sqrt{\frac{S}{\lambda Nc}}\right) > 0. \tag{A.7}$$

Taking the derivative of L^* with respect to c to obtain

$$\frac{\partial L^*}{\partial c} = 2FX - \sqrt{\frac{SF}{\lambda Nc}} \geqslant 2F\sqrt{\frac{S}{\lambda NcF}} - \sqrt{\frac{SF}{\lambda Nc}} = \sqrt{\frac{SF}{\lambda Nc}} > 0 \tag{A.8}$$

So, L^* is increasing in c as well.

In Scenario B, taking the derivative of SI^* with respect to c, we obtain

$$\frac{\partial SI^*}{\partial c} = \frac{[\lambda (M+N) cF + S]\left[p^* (1+r) - (F - cF)\right]}{4FS} > 0. \tag{A.9}$$

A.4 Proof of Proposition 5

Denote $Y = \lambda Mmp_0^{\text{budget}}$ and

$$Z = \frac{\sqrt{(M+N)G^2 + 4cF(M+N)GM} - (M+N)G}{Y}. \tag{A.10}$$

Taking the derivative of Z with respect to Y, we obtain

$$\frac{\partial Z}{\partial Y}=\frac{\frac{2cF(M+N)GY}{\sqrt{(M+N)G^2+4cF(M+N)GY}}+(M+N)G-\sqrt{(M+N)G^2+4cF(M+N)GY}}{Y^2}. \tag{A.11}$$

We have

$$\begin{cases} \frac{\partial Z}{\partial Y}>0 \ \text{ if } 0<Y<\frac{\left(M+\sqrt{M(M+N)}\right)G}{2cF} \\ \frac{\partial Z}{\partial Y}<0 \ \text{ if } Y>\frac{\left(M+\sqrt{M(M+N)}\right)G}{2cF} \end{cases} \tag{A.12}$$

This, in turn, implies

$$\begin{cases} \frac{\partial Z}{\partial m}>0 \ \text{ if } 0<m<m^* \\ \frac{\partial Z}{\partial m}<0 \ \text{ if } m>m^* \end{cases} \tag{A.13}$$

With $Z=L^{\text{budget}}+[(F+cF)-p\,(1+r)]>0$ and $SI^{\text{budget}}=\frac{\lambda M}{4cF}Z^2$, we see that both L^{budget} and SI^{budget} increase in m before reaching m^*, and decrease in m after reaching m^*.

References

[1] Almazan, A., Brown, K.C., Carlson, M., Chapman, D.A. 2004. Why constrain your mutual fund manager? Journal of Financial Economics 73, 289–321.

[2] Asquith, P., Pathak, P.A. Ritter, J.R. 2005. Short interest, institutional ownership, and stock returns. Journal of Financial Economics 78, 243–276.

[3] Blocher, J., Reed, A., Van Wesep, E.D. 2013. Connecting two markets: An equilibrium framework for shorts, longs, and stock loans. Journal of Financial Economics 108, 302-322.

[4] Chen, J., Hong, H., Stein, J.C. 2002. Breadth of ownership and stock returns. Journal of Financial Economics 66, 171–205.

[5] Cohen, L., Diether, K.B., Malloy, C.J. 2007. Supply and demand shifts in the shorting market. Journal of Finance 62, 2061–2096.

[6] D'Avolio, G. 2002. The market for borrowing stock. Journal of Financial Economics 66, 71–306.

[7] Dechow, P., Hutton, A., Meulbroek, L., Sloan, R. 2001. Short-sellers, fundamental analysis and stock returns. Journal of Financial Economics 61, 77–106.

[8] Desai, H., Ramesh, K., Thiagarajan, S.R., Balachandran, B.V. 2002. An investigation of the informational role of short interest in the Nasdaq market. Journal of Finance 57, 2263–2287.

[9] Diamond, D.W., Verrecchia, R.E. 1987. Constraints on short-selling and asset price adjustment to private information. Journal of Financial Economics 18, 277–311.

[10] Duffie, D., Gâleanu, N., Pedersen, L.H. 2002. Securities lending, shorting and pricing. Journal of Financial Economics 66, 307–339.

[11] Geczy, C., Musto, C.D.K., Reed, A.V. 2002. Stocks are special too: an analysis of the equity lending market. Journal of Financial Economics 66, 241–269.

[12] Hong, H., Stein, J.C. 2003. Differences of opinion, short-sales constraints, and market crashes. Review of Financial Studies 16, 487–525.

[13] Jarrow, R. 1980. Heterogeneous expectations, restrictions on short sales and equilibrium asset prices. Journal of Finance 35, 1105–1114.

[14] Jones, C.M., Lamont, O.A. 2002. Short sale constraints and stock returns. Journal of Financial Economics 66, 207–239.

[15] Kaplan, S., Moskowitz, T., Sensoy, B. 2013. The effects of stock lending on security prices: an experiment. Journal of Finance 68, 1891–1936.

[16] Miller, E. 1977. Risk, uncertainty, and divergence of opinion. Journal of Finance 32, 1151–1168.

[17] Nagel, S. 2005. Short sales, institutional investors, and the cross-section of stock returns. Journal of Financial Economics 78, 243–276.

[18] Scheinkman, J.A., Xiong, W. 2003. Overconfidence and speculative bubbles. Journal of Political Economy 111, 1183–1219.

[19] Sun, N., Yang, Z. 2003. Existence of equilibrium and zero-beta pricing formula in the capital asset pricing model with heterogeneous beliefs. Annals of Economics and Finance 4, 51–71.