Theory and Evidence…..

Evaluating Bundle-Size Pricing

with Simulation

by

Shijie Yu

An honors thesis submitted in partial fulfillment

of the requirements for the degree of

Bachelor of Science

Business Honors Program

NYU Shanghai

May 2017

Professor Marti G. Subrahmanyam Professor Jiawei Zhang	Professor Jiawei Zhang
Faculty Advisers	Thesis Adviser

I. Introduction

Companies selling multiple products are faced with complicated pricing decisions. In many online and offline retailing settings, for instance Taobao and supermarkets, sellers usually adopt a simple and straight-forward approach to set one price to each product. This pricing scheme is named *component pricing* (CP) in the previous literature. In addition to the most commonly seen one-product-one-price scheme, companies can alternatively adopt other effective pricing schemes. For example, *pure bundling* (PB) refers to putting all products into a bundle and setting a price to this complete bundle. On many cloud music service platforms and online streaming websites, customers have to a fixed membership fee to get access to all digital content on the platforms. *Uniform pricing* (UP) represents setting the same price for all individual products. An example would be the old Beijing subway system. No matter what the departure station and destination station are, the price is fixed at 2 RMB.

Theoretically, companies can set a price for every possible combination of their products, and this pricing scheme is called *mixed bundling* (MB). If a company is offering k products, then the total number of price to be set goes to $2^k - 1$. The advantage of *mixed bundling* is clear – Adams and Yellen (1976) show that MB strictly dominates CP and PB. Actually CP and PB can both be seen as special cases of MB. One can easily duplicate CP with MB by setting identical single product price, and attaching to all other bundles a ridiculously high price that no customers would afford. The same trick also works for replicating PB with MB. This nesting relationship guarantees MB to generate no less profit or revenue than CP and PB. Nevertheless, *mixed bundling* also has its downside. The number of pricing decisions to make grows exponentially with number of products. Once a company offers more than a few products, the pricing decision immediately becomes quite a pain to solve. Given the enormous complexity of *mixed bundling*, companies may turn to solve an alternative problem: given a constraint on number of prices, how to set up the price so as to maximize total revenue/profit from the customers?

Unfortunately, due to the lack of efficient algorithms and computing power, even this problem is extremely complicated to solve when the size of products exceeds a few. Therefore, it is more realistic to ask whether there are some other alternative pricing schemes that can capture most of the benefit of *mixed bundling*, but require much less pricing decisions to make. Bakos and Brynjolfsson (1999) give an example when tastes are independent and identically distributed and the marginal cost of products are zero, *pure bundling* can be a good approximation of *mixed bundling* when number of product goes to infinity. Chu, Leslie and Sorensen (2011) propose a new pricing scheme called *bundle-size pricing* (BSP), which "involves setting different prices for different sized bundles". For a companies offering n products, the total number of pricing decisions is also n, similar to the complexity of commonly-used *component pricing.*

Chu, Leslie and Sorensen argue that *bundle-size pricing* can be a very close approximation to mixed bundling, and tends to be more profitable than *component pricing*. Their research relies on numeric experiments to test CP, PB, BSP and MB performance under a broad range of demand and cost scenarios. Their experiment results show that BSP performs better than CP if increasing number of goods. Correlation in tastes and demand asymmetry have more complicated influences on BSP performance relative to CP.

This thesis aims to follow on Chu, Leslie and Sorensen's effort to study the performance of BSP relative to other alternative pricing schemes. The main focus is how factors like number of products, correlation among customers' tastes and demand asymmetry would impact BSP performance relative to MB and CB. I will conduct numeric experiments to simulate a wide range of consumers' taste distribution, calculate the optimal revenue with BSP, CP and MB. Then linear regression is utilized to analyze the experiment results. The experiment results suggest that BSP on average can achieve 99 % of the optimal MB value, and outperform CP in more than 94% of all cases. This evidence strongly supports BSP to be a good alternative for MB without setting too many prices. In addition, the linear regression indicates that BSP would have better performance relative to CP as the number of products offered increases, when the correlation in consumers' taste is negative, and when the demand asymmetry is small.

II. Methodology

This section will include a detailed introduction of the underlying problem that the simulation experiments are supposed to address, and the steps I take to come up with the results.

A company offering k products to the market wants to maximize its total revenue from selling its products. The company somehow knows the customers' willingness-to-pay distribution for each of the company's products, which, in my numeric experiments, are generated through Monte Carlo simulation. Similar to Chu, Leslie and Sorenson's work, I adopt the standard assumptions in the bundling literature:

1. The company is a monopolist;
2. Consumers purchase one or zero units of each product;

3. Consumers' valuations for a bundle equal the sum of their valuations for the bundle's component products (so products are neither complements nor substitutes);
4. There is no resale; and
5. Free disposal is allowed (so negative willingness-to-pay will be treated as zero).

With these assumptions, the company is able to accurately predict customers' purchase decision given a set of prices, and thus compute the revenue. The company desires to know what are the optimal prices and the corresponding revenue under each of the pricing scheme. Table 1 shows the three pricing schemes I want to evaluate in this thesis. *Bundle-size pricing*, as a relatively young and exotic pricing scheme, is the major focus of this study. *Mixed Bundling* is brought into this study because it provides the optimal revenue that the company can possibly achieve through any bundling pricing schemes. Comparing the relative performance of BSP to MB reflects whether BSP can be a good approximation to the complex MB, and how much tradeoff the company has to take to make much fewer pricing decisions. On the other hand, c*omponent pricing*, probably the most frequently used pricing schemes in real life situations, requires the same level of pricing complexity as BSP does. Comparing these two schemes under numerous taste scenarios can offer some insights to what taste factors may favor BSP, and what others may favor CP. In addition, this may also implicate some real life situations where BSP may product higher revenue over CP.

Pricing Scheme	# of prices	Description
Component Pricing	k	Each individual product sold at a different price.
Bundle-size Pricing	k	Price depends on number of purchased products.
Mixed Bundling	$2^k - 1$	Different price for all possible combination of products.

Table 1 Alternative Pricing Schemes

In order to perform a comprehensive analysis of the relative performance of BSP, I simulate a wide range of taste scenarios, optimize the three pricing schemes in each scenario, and then examine the performance of the three pricing schemes. In the existing literature, there are several distributions that are frequently used to model consumers' willingness-to-pay. Table 2 exhibits the five distributions that I test in this thesis. Furthermore, for each distribution, the parameters can take any number within a pre-defined range. Compared to Chu, Leslie and Sorenson's prior work, I expand the parameter value range so as to incorporate more extreme scenarios into my analysis.

Distribution	Range of Parameter Values
Uniform	Customers' willingness-to-pay is uniform on $[0, a_k]$, with a_k between 0.3 and 6
Normal	Customers' willingness-to-pay follows a normal distribution, with constant variance 0.25 and mean between -1 and 4
Normal (v)	Customers' willingness-to-pay follows a normal distribution, with constant mean 0 and variance between 0.25 and 1.75
Lognormal	Customers' willingness-to-pay follows a lognormal distribution, with constant variance 0.25 and mean between -1.5 and 2
Exponential	Customers' willingness-to-pay follows an exponential distribution with mean between 0.2 and 2

Table 2 Alternative Willingness-to-pay Distributions

In addition to various taste distributions, I also consider different number of products. As number of products rises, the complexity of solving the price optimization problem also increases dramatically. Constrained by limited computing power, I only test number of products from 2 to 4. And each number of products involve five different taste distribution. Therefore, combinations of number of products and taste distribution results to 15 general scenarios, and each single scenario involves a few independent simulations and optimizations.

In each simulation, I generate a set of arbitrary distribution parameters from the pre-defined range, then simulate 10,000 random customers' willingness-to-pay accordingly. Next, these virtual customer demand are fed as raw input into an optimization model, of which the decision variables are product and bundle prices under BSP, CP and MB respectively, and the objective is to maximize total revenue from these 10,000 customers. After repeating this simulation-optimization cycle for all taste scenarios, I obtain a large cross-sectional dataset that records the relative performance of BSP as opposed to MB and CP under different taste distributions and number of products. This dataset provides the foundation for my later result analysis and interpretation phase.

III. Model and Results

In order to summarize the general pattern from a large quantity of experiment results across various distribution parameters, I completed a multiple linear regression on the experiment results. The first regression aims to study how the performance of BSP relative to CP (*bsp_cp*) is influenced by other factors. The relative performance is measured by the optimal BSP revenue divided by the optimal CP revenue in each experiment, and is set to be the dependent variable in the first regression. Similarly, the performance of BSP relative to MB (*bsp_mb*), measured by the optimal BSP revenue divided by the optimal MB revenue in each experiment, is set to be the dependent variable of the second regression. Note that *bsp_mb* is less than or equal to one due to the nesting relationship.

Table 3 shows the descriptions of all the independent variables in the regression. The same group of independent variables applies for both regressions. In the experiments, I consider three number of goods scenario (k=2, 3, 4) and use dummy variables to mark these different scenarios. To avoid perfect collinearity, the 2-product scenario is set as the base case, and thus omitted from the regression. The estimated coefficient of these dummies can be interpreted as how the performance of BSP relative to CP would change if number of products increases. *aMean* and *aVar* are two measurements for demand asymmetry, where *aMean* describes the level of difference of consumers' expected willingness-to-pay among all products offered. For example, the *aMean* for a movie ticket is relatively small, while the *aMean* for laptops and software can be extremely high. This concept more or less depicts to what extent the intrinsic value of the products differ from each other. *aVar*, on the other hand, contains the remaining part of demand asymmetry caused by discrepant willingness-to-pay among consumers. The four dummy variables for distributions are brought in to control the impact that different distribution assumptions may shadow on the relative performance of BSP. The uniform distribution is set as the base case, and thus omitted from the regression.

Independent Variable	Value	Description
k3	Binary	= 1 if the company offers 3 products, 0 otherwise
k4	Binary	= 1 if the company offers 4 products, 0 otherwise
aMean	Numerical	Variance of average valuation of each product offered
aVar	Numerical	Variance of valuation variance of each product offered
correlation	Numerical	Correlation among valuation of each products
normal	Binary	= 1 if the taste distribution is normal, 0 otherwise
normal (v)	Binary	= 1 if the taste distribution is normal (v), 0 otherwise
lognormal	Binary	= 1 if the taste distribution is lognormal, 0 otherwise
exponential	Binary	= 1 if the taste distribution is exponential, 0 otherwise

Table 3 Independent Variables in the Regression

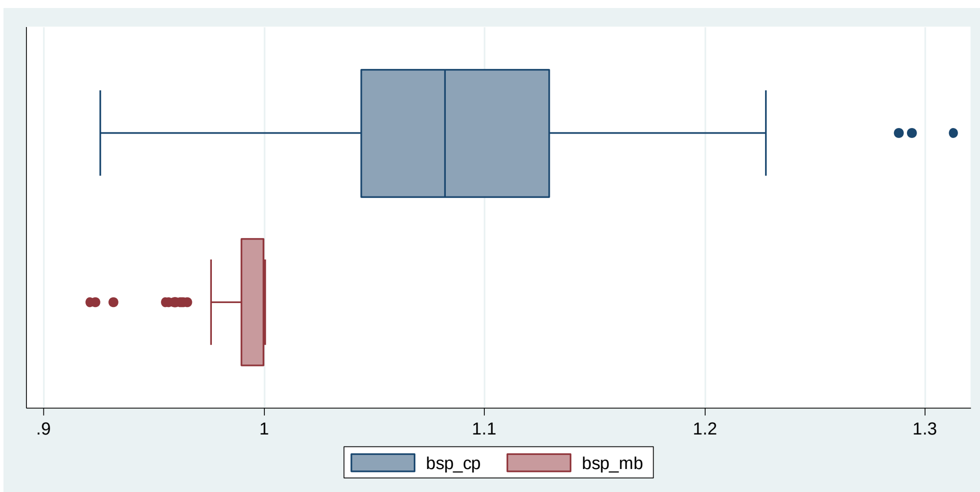

Figure 1 Distributions of BSP Performance Relative to CP and MB

Figure 1 displays a box plot of the distributions of BSP performance relative to CP and MB respectively. The upper adjacent value, 75th, 50th, 25th, and the lower adjacent value are shown in the plot. The dots represent outside values occurred in the experiments. The red box indicates that under my customer taste assumptions, BSP is a close approximation to MB in most cases. On average, BSP can achieve 99% of the optimal MB revenue, and more than 74% experiments are beyond the average level. The experiment result also reveals that BSP tends to be more profitable than CP. CP achieves higher revenue than BSP in merely 5.8% of all experiments. On average, BSP can obtain 8.7% more revenue compare to CP. Compared to the experiment result shown in Chu, Leslie and Sorensen's work, the overall performance of BSP in my research is slightly better. The reason might be their work takes into account cost structure assumptions, which are not implemented in my model. When the marginal cost of the products are high, the BSP profit relative to both BSP and CP drops (Chu et al 2011), and therefore dragging down the average BSP profit relative to BSP and CP. Nevertheless, the tiny difference does not undermine the core conclusion that BSP can be a much simpler alternative to MB without sacrificing much revenue, and meanwhile, with the same number of prices to set, BSP can obtain higher revenue than CP in most cases.

Source	SS	df	MS			
				Number of obs	=	104
				F(9, 94)	=	31.13
Model	.378660351	9	.042073372	Prob > F	=	0.0000
Residual	.127034475	94	.001351431	R-squared	=	0.7488
				Adj R-squared	=	0.7247
Total	.505694825	103	.004909658	Root MSE	=	.03676

bsp_cp	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
k3	.0288873	.0090988	3.17	0.002	.0108214	.0469533
k4	.0926602	.0108089	8.57	0.000	.0711989	.1141215
aMean	-.003763	.0107619	-0.35	0.727	-.0251311	.017605
aVar	-.0485414	.0131754	-3.68	0.000	-.0747014	-.0223814
correlation	-.1224515	.0345806	-3.54	0.001	-.1911121	-.0537908
normal	-.0261759	.0123719	-2.12	0.037	-.0507407	-.0016112
normalv	-.0748247	.013314	-5.62	0.000	-.1012599	-.0483896
lognormal	.0116052	.0130679	0.89	0.377	-.0143414	.0375519
exponential	.0866514	.0122339	7.08	0.000	.0623606	.1109422
_cons	1.08622	.0093768	115.84	0.000	1.067602	1.104838

Table 4 ANOVA and Estimated Coefficients (BSP/CP)

Source	SS	df	MS			
Model	.00874805	9	.000972006	Number of obs	=	104
Residual	.020122454	94	.000214069	F(9, 94)	=	4.54
				Prob > F	=	0.0001
Total	.028870505	103	.000280296	R-squared	=	0.3030
				Adj R-squared	=	0.2363
				Root MSE	=	.01463

bsp_mb	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
k3	-.0080408	.0036213	-2.22	0.029	-.015231	-.0008506
k4	.0066407	.0043019	1.54	0.126	-.0019009	.0151822
aMean	.007892	.0042832	1.84	0.069	-.0006124	.0163964
aVar	-.0152239	.0052438	-2.90	0.005	-.0256355	-.0048123
correlation	.0000655	.013763	0.00	0.996	-.0272612	.0273922
normal	-.0021949	.004924	-0.45	0.657	-.0119716	.0075817
normalv	-.0101435	.0052989	-1.91	0.059	-.0206646	.0003777
lognormal	-.0044609	.005201	-0.86	0.393	-.0147876	.0058658
exponential	.0106094	.0048691	2.18	0.032	.0009418	.0202771
_cons	.9939163	.003732	266.33	0.000	.9865064	1.001326

Table 5 ANOVA and Estimated Coefficients (BSP/MB)

Number of Goods

Table 4 and table 5 exhibit the regression results of the BSP performance relative to BSP and CP respectively. The regression results by taste distribution are also attached in the appendix. The results suggest that increasing the number of goods tend to favor BSP over CP. Why is that? Assuming that consumers' willingness-to-pay follows a certain distribution for each product, it is common that the consumers have high willingness-to-pay for some products and low willingness-to-pay for other products. Bundles tend to balance out these variations in willingness-to-pay for single products, resulting to lower variance among consumers' willingness-to-pay for bundles. When the variance of willingness-to-pay is low, the same price would be accepted by more consumers compared to high variance of willingness-to-pay, and therefore increases the total income. As the size of the bundle increases, the balancing-out effect becomes increasingly significant, so bundling-type pricing strategies (BSP and MB) are inclined to be more profitable than CP.

The impact on the revenue of BSP compared to MB is, however, insignificant. Nevertheless, the insights from the optimal price more or less imply why BSP can be a good approximation to MB. According to the experiment results, the optimal price in BSP closely follows the highest MB price of the same bundle size. Due to the balancing-out effect, the optimal MB price of large-size bundles tend to be close to each other. In this sense, BSP approximates large MB bundles nicely. Meanwhile, the experiment results also suggest that more customers tend to purchase large-size bundles (the full bundle takes up the largest market share). Therefore, it's not surprising that BSP can perform so well.

Correlation

Table 4 also shows that BSP performs better with respect to CP in negatively-correlated willingness-to-pay scenarios. Holding other factors constant, CP revenue is insensitive to the correlation among tastes in that the optimal price of each product is determined exclusively by the overall taste distribution of the product. This implies that BSP revenue has to be influenced by the correlation in consumers' tastes among products.

It is already discussed in the number of goods subsection that BSP and MB benefit from the balancing-out effect on consumers' tastes on bundles. Assuming a two product situation and independent taste between the two products, the variation in consumers' taste for the two-product bundle is likely to be somehow smaller than for the single product with larger variance in taste. If the correlation in taste is now negative instead of zero, consumers tend to have high willingness-to-pay on one of the product and low willingness-to-pay on the other product. Thus, the overall willingness-to-pay distribution for the two-product bundles tends to have smaller variance rather than the independent taste scenario. On the contrary, consumers' willingness-to-pay of the two products tend to go to the same extreme in positive-correlated taste scenarios. In this case, the taste distribution of the two-product products is expected to have larger variance than the independent case scenario. Since the same price can capture higher revenue when tastes are less varied among consumers, the lower the correlation is, the higher the BSP revenue is predicted to be.

Demand Asymmetry

In this paper, demand asymmetry is separated into two parts – the asymmetry caused by products' value (*aMean*) and the asymmetry caused by variation in consumers' taste (*aVar*). The regression result shows that *aMean* does not have a significant impact on BSP revenue relative to CP, but a slightly positive effect on BSP revenue relative to MB. On the other hand, aVar has a negative impact on BSP revenue relative to both CP and MB.

The result that BSP revenue relative to CP is not influenced by *aMean* coincides with Chu, Leslie and Sorensen's experiment results. They provide an explanation with a two-consumer-two-product example where customer A have willingness-to-pay 2 for product 1 and 0 for product 2, and customer B have willingness-to-pay 1 for product 2, and 0 for product 1. In this case, CP can achieve 3 in revenue while BSP can only achieve 2, in another words CP can obtain 50% more revenue over BSP. If we change customer A's willingness-to-pay for product 1 to 1, then BSP would perform as well as CP. If changed to 10, then CP would only perform 5% better than BSP. This toy example indicates that BSP can approximate the CP revenue when the variance in products' value is extremely small or quite high.

That the BSP performance relative to MB improves as the asymmetry among products' value decreases is not as obscure. It can be observed from the experiment results that the optimal BSP prices are close to the highest prices across all sizes of bundles under MB. Therefore, when the taste differs dramatically among products, the optimal BSP price would be a lot larger than the average willingness-to-pay for some bundles, especially those filled with products of relatively low value. In this sense, BSP would sacrifice those "cheap MB bundle consumers". When the asymmetry of product value is low, consumers' willingness-to-pay for different bundles of the same size concentrate more, and therefore BSP sacrifice less "cheap bundle consumers".

IV. Conclusion

This paper studies the BSP revenue with respect to MB and CP by using simulation and optimization methods. In order to make the result more robust to different willingness-to-pay distributions and corresponding parameters, the simulation process incorporates a wide range of taste distributions as well as correlation among consumers' willingness-to-pay. In each experiment, the optimal revenue under the three pricing strategies is calculated, and provides the input for analyzing the general performance of BSP relative to the other two pricing strategies.

This thesis has two major findings. BSP is a very good approximation to MB, and requires much less pricing decisions to make. In my experiments, BSP can obtain around 99% of the optimal MB revenue on average. Meanwhile, BSP also outperforms CP, which requires the same number of pricing decisions as BSP, in more than 94% of all cases. This result suggests that BSP, a newly-proposed pricing strategy that scarcely used in the real life, actually has great potential to help companies achieve higher revenue while maintaining or even reducing the number of pricing decision to make.

On the other hand, this paper also investigates which factors may influence BSP revenue relative to CP and BSP with a linear regression based on all experiment results. The regression indicates that the following factors favor BSP relative to CP:

(1) Increasing number of goods offered;

(2) Negative correlation among consumers' willingness-to-pay for products;

(3) Low variance among consumers' taste among products.

Appendix

	Exponential	Lognormal	Normal	Normal (v)	Uniform	All Combined
K=3	0.488 (0.014)	-0.025 (0.019)	0.008 (0.015)	0.049 (0.027)	0.072 (0.019)	0.042 (0.015)
K=4	0.148 (0.017)	0.072 (0.028)	0.070 (0.018)	0.036 (0.023)	0.102 (0.022)	0.094 (0.018)
aMean	-0.277 (0.104)	-0.006 (0.035)	-0.003 (0.013)	-1.893 (1.953)	-1.114 (0.055)	-0.001 (0.012)
aVar	0.067 (0.042)	0.010 (0.067)	-17.568 (14.784)	-0.057 (0.021)	0.050 (0.051)	-0.026 (0.015)
correlation*	-	-	-0.116 (0.028)	-	-	-0.123 (0.057)
Constant	1.156 (0.013)	1.107 (0.016)	1.076 (0.014)	1.035 (0.018)	1.085 (0.015)	1.073 (0.010)
R^2	0.923	0.505	0.600	0.618	0.732	0.284

Table 6 Regression Analysis of BSP/CP

	Exponential	Lognormal	Normal	Normal (v)	Uniform	All Combined
K=3	-0.010 (0.004)	-0.024 (0.009)	-0.015 (0.003)	0.021 (0.019)	-0.001 (0.008)	-0.007 (0.004)
K=4	-0.001 (0.005)	-0.005 (0.013)	-0.004 (0.004)	0.022 (0.017)	0.015 (0.009)	0.007 (0.005)
aMean	-0.011 (0.027)	0.033 (0.017)	0.004 (0.003)	-1.958 (1.405)	0.011 (0.023)	0.007 (0.003)
aVar	0.001 (0.011)	-0.049 (0.032)	8.788 (3.403)	-0.013 (0.015)	-0.032 (0.021)	-0.013 (0.004)
correlation*	-	-	-0.003 (0.007)	-	-	0.000 (0.015)
Constant	1.002	0.983 (0.008)	0.991 (0.003)	0.987 (0.013)	0.997 (0.006)	0.992 (0.003)
R^2	0.388	0.410	0.541	0.388	0.285	0.167

Table 7 Regression Analysis of BSP/MB

REFERENCES

Adams, William James, and Janet L. Yellen. "Commodity Bundling and the Burden of Monopoly." *The Quarterly Journal of Economics*, vol. 90, no. 3, 1976, p. 475., doi:10.2307/1886045.

Bakos, Yannis, and Erik Brynjolfsson. "Bundling Information Goods: Pricing, Profits, and Efficiency." *Management Science*, vol. 45, no. 12, 1999, pp. 1613–1630., doi:10.1287/mnsc.45.12.1613.

Chu, Chenghuan Sean, et al. "Bundle-Size Pricing as an Approximation to Mixed Bundling." *American Economic Review*, vol. 101, no. 1, 2011, pp. 263–303., doi:10.1257/aer.101.1.263.