PDE/ANALYSIS SEMINAR

Topic: **Numerical Homogenization without Scale Separation**

Speaker: **Lei Zhang, Shanghai Jiao Tong University**

Time: 16:15-17:15, 19 March 2014

Venue: Room 371, Geography Building, 3663 Zhongshan Road North, Shanghai
(华东师范大学中山北路校区，地理楼 371 室)

ABSTRACT OF THE TALK

The field of numerical homogenization concerns the numerical approximation of the solution space of, for example, divergence form elliptic equation with L^∞ coefficients by a finite-dimensional space. This problem is motivated by the fact that standard methods such as finite-element method with piecewise polynomial elements can perform arbitrarily badly for PDEs with rough coefficients. Some numerical homogenization methods are developed from classical homogenization concepts such as periodic homogenization and scale separation, however, one of the main objectives of numerical homogenization is to achieve a numerical approximation of the solution space of the equation with arbitrary rough coefficients. For problem with nonseparable scales, we have proposed the method of harmonic coordinates for scalar elliptic equation in 2D [4]. In [2] the transfer property of the flux-norm is introduced to identify the global basis. In [5], we conclude the strong compactness of the solution space, which guarantees the existence of accurate finite-dimensional approximation space as long as the right hand side is not too singular. Now the name of the game becomes how to achieve such a finite-dimensional space with optimal convergence rate and least cost, namely, the space with best localized basis, which we discussed in [5, 6]. For development in this direction, please also see [1, 3].

References

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BIOGRAPHY

Lei Zhang is Associate Professor of Mathematics at the Shanghai Jiao Tong University. He received his Ph.D. in Applied and Computational Mathematics from California Institute of Technology in 2007. He is especially interested in multiscale analysis, modeling and simulation. His current focus is numerical homogenization and atomistic/continuum coupling.