NYU-ECNU Institute of Mathematical Sciences at NYU Shanghai

WEEKLY SEMINAR

Topic: **Counting Roots of Polynomials in a Prime Field**

Speaker: **Dr. Kenneth Ward, Visiting Professor and Mathematics Coordinator of NYU Shanghai**

Time: 15:30-16:30, 25 September 2013

Venue: Room 153, Geography Building, 3663 Zhongshan Road North, Shanghai

(华东师范大学中山北路校区，地理楼 153 室)

ABSTRACT OF THE TALK

Weyl's equidistribution criterion for Heilbronn's exponential sum

$$S(a) = \sum_{n=1}^{p} \exp\left(2\pi i \frac{an^p}{p^2}\right)$$

was studied by Heilbronn and Davenport who gave a type of $o(p)$ bound in 1936. Heath-Brown later gave a bound of $O\left(p^{\frac{11}{12}}\right)$ using Stepanov's method (with many others giving partial or better results in specific cases using similar methods). Observations of Bombieri and Odoni say that on average such sums should be $O\left(p^{\frac{1}{2}}\right)$. In the case of $S(a)$, this is linked to the number of roots in F_p of the truncated logarithm

$$L(X) = X + \frac{X^2}{2} + \cdots + \frac{X^{p-1}}{p-1},$$

which is known to be $O\left(p^{\frac{2}{3}}\right)$. The function $L(X)$ is special in that it satisfies a differential equation (mod X^p) of degree one. There is no known work on this problem for differential equations of degree greater than one. For differential equations of higher degree, I can prove that the truncated Bessel function admits the bound $O\left(p^{\frac{6}{7}}\right)$ and some weak $o(p)$ bounds on polylogarithms. I ask how one might prove similar results for Fuchsian differential equations in general and what implications this may have for elliptic curves via Hasse's invariant.

BIOGRAPHY

Kenneth Ward has held positions in teaching and research at the University of Chicago and Oklahoma State University. His primary research interest is arithmetic geometry and its applications.