NYU-ECNU
Institute of Mathematical Sciences
at NYU Shanghai

ANALYSIS/PDE SEMINAR SERIES

TOPIC: **Wellposedness of Variable-coefficient Conservative Fractional Elliptic Differential Equations and Numerical Methods**

SPEAKER: **Professor Danping Yang, Department of Mathematics, ECNU**

TIME: 1:45-2:45, 18 December 2014

VENUE: Room 357, Geography Building, 3663 Zhongshan Road North, Shanghai
(中山北路校区，地理楼 357 室)

ABSTRACT OF THE TALK

Fractional diffusion equations describe various phenomena exhibiting anomalous diffusion that cannot be modeled accurately by second-order diffusion equations. Fractional differential equations raise many mathematical difficulties that have not been encountered in the analysis of second-order differential equations. There are two properties of fractional differential operators that make the analysis of fractional differential equations more complicated than that for classical second-order differential equations. These are (i) the fractional differential operators are nonlocal operators, and (ii) the adjoint of a fractional differential operator is not the negative of itself. The wellposedness of a Galerkin weak formulation to fractional elliptic differential equations with a constant diffusivity coefficient and the error analysis for corresponding finite element methods were proved previously. Many subsequent works were carried out to extend the analysis to other numerical methods. A constant diffusivity coefficient has been assumed in all these works.

In this talk, we present a counterexample which shows that the Galerkin weak formulation loses coercivity in the context of variable-coefficient conservative fractional elliptic differential equations. Hence, the previous results cannot be extended to variable-coefficient conservative fractional elliptic differential equations. We adopt an alternative approach to prove the existence and uniqueness of the classical solution to the variable-coefficient conservative fractional elliptic differential equation and characterize the solution in terms of the classical solutions to second-order elliptic differential equations. Furthermore, we derive a Petrov-Galerkin weak formulation to the fractional elliptic differential equation. We prove that the bilinear form of the Petrov-Galerkin weak formulation is weakly coercive and so the weak formulation has a unique weak solution and is well posed. Finally, we outline potential application of these results in the development of numerical methods for variable-coefficient conservative fractional elliptic differential equations.

BIOGRAPHY

Danping Yang is Professor of Department of Mathematics at East China Normal University. His research fields are numerical methods for partial differential equations, domain decomposition and parallel algorithm, numerical methods for optimal control problem, numerical modeling and simulation for multi-scale problem and numerical analysis.